INVESTIGATING SENIOR SECONDARY SCHOOL STUDENTS' BELIEFS ABOUT FURTHER MATHEMATICS IN A PROBLEM-BASED LEARNING CONTEXT

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Abstract. The study investigated the effect of problem-based learning (PBL) on senior secondary school students' beliefs about Further Mathematics in Nigeria within the blueprint of pre-test-post-test non-equivalent control group quasi-experimental design. Intact classes were used and in all, 96 students participated in the study (42 in the experimental group taught with the PBL and 54 in the control group taught using the Traditional Method (TM)). One research instrument tagged Beliefs about Further Mathematics Questionnaire (BFMQ, Cronbach alpha (α)=.86) was developed and used for the study and data collected were analysed using the descriptive statistics of mean and standard deviation which served as precursor to testing the null hypothesis for the study using an independent samples t-test and analysis of variance. Results showed that participants held strong beliefs about further mathematics and there was a statistically significant difference in the mean post-treatment scores on BFMQ ($t=-6.22$, $p=.000$ for t-test) and ($F_{(1,95)}=38.49$; $p<.001$ for ANOVA) between students exposed to the PBL and those exposed to the TM, in favour of the PBL group. Based on the results, the study recommended that PBL should be adopted as an instructional strategy for promoting meaningful learning in and enhancing beliefs about further mathematics and efforts should be made to integrate the philosophy of PBL into the preservice teachers' curriculum at the teacher-preparation institutions in Nigeria.

Keywords: problem-based learning, traditional method, further mathematics, beliefs about further mathematics

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Introduction

Research on beliefs dates a long way back and the importance of investigating teachers' and students' mathematical beliefs in mathematics education research since the last three decades has been highlighted in several studies along different perspectives (Handal, 2003; Kagan, 1992; Leder, Pehkonen, & Törner, 2002; Roesken, 2011). Several researchers have highlighted the role of general epistemological beliefs in school achievement (Schommer, 1993; Muis & Franco, 2009) and Hofer (2002) provided a list of phenomena in epistemological beliefs that has attracted concern from educational researchers to include "the definition of knowledge, how knowledge is constructed, how knowledge is evaluated, where knowledge resides, and how knowing occurs". Beliefs are paramount given that they can generate psychological domains of behaviour and have been studied in relation to attitudes, emotions, and values (Pehkonen, 2004; Csíkos, 2011). Goldin (2002) gave a general distinction among emotions, attitudes, beliefs and values in mathematics education: (1) *emotions* (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context), (2) *attitudes* (moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition), (3) *beliefs* (internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured), and (4) *values, ethics, and morals* (deeply-held preferences, possibly characterized as "personal truths", stable, highly affective as well as cognitive, may also be highly structured). As a derivative of the general epistemological beliefs, mathematical beliefs can determine how one chooses to mentally construct the whole idea of mathematics and there is an agreement on the multidimensional characteristic of mathematical beliefs (De Corte et al., 2002) with different factors found in empirical studies (Andrews et al., 2007). Muis et al. (2011) in an empirical study revealed the two-faceted nature of (graduate) students' beliefs in statistics.

Beliefs are personal principles, constructed from experience that an individual employs often unconsciously to interpret new experiences and information and to guide action (Pajares, 1992). Cobb (1986) defined beliefs as an individual personal assumption about the nature of reality. The importance of beliefs in the life of a student is stressed again because these assumptions constitute the goal-oriented activity. Beliefs play a significant role in directing human's perceptions and behaviour. In learning environments, students' belief might propagate the idea for achievements and smoothness of learning. In the Mathematics learning process, students' belief about the nature of Mathematics and factors related to the learning are two components that always concern Mathematics educators. Fennema & Sherman (1978) reported that middle school and high school students who achieved higher scores on tests of mathematical achievement perceived mathematics to be more useful than lower-achieving students did. Schreiber (2000) studied attributions associated with successful achievement and found the more a student believed that success in mathematics was caused by natural ability, the higher the test score.

Several researchers (Amarto & Watson, 2003; Chick, 2002; Morris, 2001) have reported that pre-service teachers do not always have the conceptual understanding of the mathematics content they will be expected to teach. Aldridge & Bobis (2001) reported a change in beliefs about mathematics towards a more utilitarian and problem solving perspective because of a university education program. Schuck & Grootenboer (2004) stated that research 'on the beliefs of student teachers has found that prospective primary school teachers generally hold beliefs about mathematics that prevent them from teaching mathematics that empower children'. House (2006) conducted a study to compare the relationship of mathematics beliefs and achievement of elementary school students in Japan and the United States based on the Third International Mathematics and Science Study (TIMSS). The study revealed that students in Japan scored above the International averages. Chen & Zimmerman (2007) compared the students' mathematical beliefs between America and Taiwan and they found that the Taiwanese surpassed American in mathematics achievement. Their results supported the finding from TIMSS (1995)[1)] report on the International comparison of the two countries. They concluded that there were more similarities in mathematics beliefs regarding mathematical competence of Taiwanese and American students. The result of the study showed that students from both countries have undistinguishable beliefs in the difficulty level of mathematics questions especially the easy and difficult mathematics items.

Op't Eynde & De Corte (2003) conducted a research on mathematics beliefs among Belgium secondary school students and the findings showed that most students believe that mathematics is an interesting discipline to be learnt. They also found that there is a significant difference among students in terms of their mathematics ability. The higher students have more positive mathematics belief particularly in the functions of teachers in teaching mathematics compared to those low achievers. Given the important role of the learner in the education process, it appears quite natural to study in-depth his or her personal philosophies about mathematics. Beliefs have shown to affect how students learn and what they want to learn (Şahin, 2009a) and helping them to attain more expert-like beliefs within the context of constructivist instructional strategies can foster optimal learning. One constructivist instructional strategy that is currently the focus of more research is problem-based learning (PBL). This strategy has enjoyed world-wide research and has been in some cases revealed to promote students' social skills, motivation, and interest in the subject matter. The current study focused on secondary school students of further mathematics with a view of determining the impact of PBL on beliefs. Originally developed at McMaster University over four decades ago (Barrows & Tamblyn, 1976, 1980), PBL has gained prominence as an interactive instructional strategy in medicine, engineering, and education (Edens, 2000; Edwards & Hammer, 2004; Eldredge, 2004; Fink et al., 2002; Jones, 2006; Şahin, 2009; Selcuk & Şahin, 2008; Stonyer & Marshall, 2002). It has equally been used in physical science subjects like physics (Duch, 1996; Iroegbu, 1998; Raine & Collett, 2003; Şahin, 2007; Şahin & Yörek, 2009; Şahin, 2009b). While PBL has been researched into and very well documented as an effective strategy in one hand for enhancing students' learning outcomes in varied school subjects across many countries in Europe, America, Asia, and Africa (Albanese & Mitchell, 1993; Berkson, 1993; Colliver, 2000; Norman & Schmidt, 1992, 2000; Major & Palmer, 2001; Prince, 2004; Vernon & Blake, 1993) on the other hand, the effectiveness of PBL in the educational classroom is somewhat fraught with mixed conclusions. For instance, Albanese & Mitchell (1993) concluded that problem-based instructional approaches were less effective in teaching basic science content (as measured by Part I of the National Board of Medical Examiners exam), whereas Vernon & Blake (1993) reported that PBL approaches were more effective in generating student interest, sustaining motivation, and preparing students for clinical interactions with patients. Mixed results were also observed in the studies by Moust et al. (2005) and Prince (2004) in which the latter maintained that it is difficult to conclude if it is better or worse than traditional curricula, and that "it is generally accepted …that PBL produces positive student attitudes" whereas the former concluded that PBL has a positive effect on the process of learning as well as on learning outcomes. According to Major & Palmer (2001) students in PBL courses often report greater satisfaction with their experiences than non-PBL students whereas Beers (2005) demonstrated no advantage in the use of PBL over more traditional approaches.

These mixed results regarding the effectiveness of PBL in specific contexts warrant further studies and more importantly the effectiveness of PBL in Nigerian classrooms in different school subjects is not yet popular. A few studies investigated the use of PBL in the Nigerian classrooms with attention focused on the cognitive domain (Iroegbu, 1998) and physics being the favourite school subject. In Nigeria, there are overwhelming evidences in support of the general decrease in students' enrolment in further mathematics and this has generated a relentless search for alternative ways of instruction among mathematics educators and further mathematics teachers. Evidence suggests that the high attrition rate in most physical science subjects and concomitant poor performance in the subjects at the senior secondary school level could be reduced to the barest minimum with the implementation of the PBL (Abraham et al., 2012; Burch et al., 2007). Judging by the effectiveness of PBL in medicine and engineering, particularly in motivating students, Şahin (2009a) made a case for the adoption of PBL as an alternative instructional strategy in addressing decreasing students' enrolment in physics. PBL takes place in the context of the real world (Savery & Duffy, 1995) and is appropriate for mixed-ability teaching (Şahin, 2009b) and when the desire is to develop skills in group work. The traditional methods of teaching have been criticised for not being suitable for effecting mixed ability teaching and developing skills in group work and students in their day to day interaction with the environment are bathed with complex problems that demand problem solving skills and working cooperatively for solutions. The Nigerian further mathematics content is woven round complex problems that are ill structured and teaching the content demands students taking up an active role in which they would be able to construct their own knowledge of further mathematics contents cooperatively in an inquiry, problem-solving based context. PBL has been found effective and more amenable in teaching and combating ill structured problems (Sungur & Tekkaya, 2006). While literature is replete with the efficacy of PBL in students' academic achievement, conceptual development, and attitudes towards science courses (Akınoğlu & Tandoğan, 2007), researches into its efficacy in students' beliefs are scarce with neutral and negative findings. Şahin (2009b) investigated the correlations of PBL and traditional students' course grades, expectations and beliefs about physics and selected student variables in an introductory physics course in engineering faculty. PBL and traditional groups were found to be no different in their responses to the Maryland Physics Expectations Survey (MPEX) and in their physics grades. In addition, students who showed effort and studied hard tended to obtain higher physics grades. Şahin (2009b) in a pretest-posttest quasi-experimental study of the effect of instructional strategy manipulated at two levels; modular-based active learning (problem-based learning [PBL]) method and traditional lecture method on university students' expectations and beliefs in a calculus-based introductory physics course measured with the Maryland Physics Expectations (MPEX) survey revealed that average favourable scores of both groups on the MPEX survey were substantially lower than that of experts and that of other university students reported in the literature. He maintained that students' favourable scores on the MPEX survey dropped significantly after one semester of instruction and both PBL and traditional groups displayed similar degree of ‘expert’ beliefs. He concluded that university students’ expectations and beliefs about physics and physics learning deteriorated as a result of one semester of instruction whether in PBL or traditional context. Research on students’ beliefs is important since beliefs affect motivation (Hofer & Pintrich, 1997), influence students’ selection of learning strategies (Edmonson, 1989; Schommer et al., 1992) enable students’ to gain conceptual learning and understanding (Songer & Linn, 1991; May & Etkina, 2002) and solve problems (Hammer, 1994). In view of the above background, this study investigated the effect of PBL on senior secondary school students’ beliefs about further mathematics in Nigeria.

Research questions

Based on the problem aforementioned problem, this study provided answers to the following research questions: (1) what are the beliefs of senior secondary school students towards further mathematics teaching and learning; (2) will there be any significant difference in the post-treatment scores on Beliefs about Further mathematics questionnaire (BFMQ) between students exposed to the PBL and those exposed to the TM?

Null hypothesis

The following null hypothesis was tested in this study at.05 level of significance.

Ho_1: There is no statistically significant difference between the post-treatment scores on BFMQ of students exposed to the PBL and those exposed to the TM.

Method

Research design

The model of inquiry adopted for this study was a quantitative method (Creswell & Plano Clark, 2011) described as a systematic empirical investigation of social phenomena via statistical, mathematical or computational techniques (Bergma, 2008) within the blueprint of quasi-experimental design using pretest-posttest non-equivalent control groups.[2] The quasi-experimental design allows identification of variables (Blaxter et al., 1996) in the study. The quasi-independent variable-instructional strategy was manipulated at two levels (PBL & TM) and answering the research questions for the study required data that allowed assessment of the extent to which the PBL and TM influence students' beliefs about further mathematics. This study relied on interval (scores on Beliefs about Further Mathematics Questionnaire) data as the stronger form of quantification (Okpala et al., 1993). In this study, participants do not have control over which group (control or experimental) they belonged or of receiving or not receiving the treatment based on quasi-experimental design. One inherent advantage of this design is that it is typically easier to set up than true experimental designs (Shadish et al., 2002) but lacks randomisation of subjects to treatment conditions.[2] Adopting quasi-experimental design in this study allowed the investigation of intact group in real classroom settings since it was not necessary to randomly assemble students for any intervention during the school hours so as not to create artificial conditions. Students in control and experimental groups participated in the study in their natural classroom conditions.

Population, sample and sampling method

The study was conducted in the Ijebu division of Ogun State of Nigeria. The division is made up of six out of twenty Local Government areas constituting Ogun State. The local governments are Ijebu East; Ijebu North; Ijebu North East; Ijebu Ode; Odogbolu and Ogun Waterside. Ijebu division, which is, populated predominantly by Ijebu tribe, has a population of about 816 681 out of the recorded figure of 3 751 140 for the State.[3] In the education sector, there are many primary and secondary schools owned by individuals and missionaries apart from the public ones owned by the government. For the purpose of this study, the government owned public secondary schools were considered as all others did not allow any interference in the administration of their schools. Only Ijebu-Ode Local Government out of the existing six local governments in the division was considered for the study based on the following criteria: proximity to the base of the researcher, the researcher's familiarity with the geographical terrain, and accessibility to information at the Zonal Ministry of Education.

The 319 Senior Secondary School (SSS1) year one science students (an equivalent of Grade 10) taking further mathematics at the 30 senior secondary schools in Ijebu-Ode Local Government of the Ijebu division of Ogun state constituted the target population. As stipulated in the National Curriculum for Senior Secondary Schools for Further Mathematics,[4)] further mathematics is meant for potential Mathematicians, Engineers and Scientists. Consequently, all schools that have qualified graduate mathematics teachers are expected to offer the subject to cater for science students' interest. Among the 30 schools in the local government, eight were found to be offering further mathematics. This is due to paucity of qualified graduate mathematics teachers and their non-willingness to teach the subject.

A breakdown of the total number of students taking further mathematics at the eight schools coded A - H is given in the sequel (School A has 42 students, B has 54, C has 34, D has 35, E has 41, F has 35, G has 30 and H has 48). This population was considered for the study because of the following reasons: (i) this is the class where further mathematics instruction begins in Nigeria Senior Secondary Schools; (ii) these group of students were not preparing for any immediate external examination (unlike the Senior Secondary School year three students); hence, the schools would be willing to allow them to participate in the study; (iii) the researcher was of the opinion that this level of students is mature enough to express their opinions about beliefs toward further mathematics.

In selecting schools to participate in the study, purposive sampling and simple random sampling techniques were used. Purposive sampling relies on the judgment of the researcher when it comes to selecting the units using certain criter*ia.* One of the criteria of purposive sampling technique was based on few in number of schools offering further mathematics and was considered appropriate for the study. More so, graduate teachers from other disciplines like physics and economics were found teaching FM in four schools at the time the study was conducted. Thus, the following criteria were used in selecting the schools that participated in the study. The schools were to: (i) have qualified graduate mathematics teachers who have been teaching in the school for at least three years. The three years minimum was the researcher's decision to ensure some degree of teachers' cognate experience; ii) have been presenting candidates in West African Senior School Certificate Examination (WASSCE) for at least four years consistently. The minimum of four years was the researcher's decision to ascertain that the schools have been presenting candidates in FM at external examination; (iii) has principal and mathematics teachers who would be willing to cooperate and participate in the study; (iv) are public government owned secondary schools.

Thus, four out of the eight schools emerged based on the foregoing criteria. Simple random sampling technique was used in selecting schools for the pilot and the main study. This involved writing the initials of each of the four schools on different pieces of paper and each was squeezed into a bolus on the floor. The decision was that, the first two boluses that were handpicked were tagged schools for pilot study whereas the remaining two went for the main study. A young lady was asked to handpick two boluses at a time. Thus, two schools each emerged for the pilot study and the main study. However, in the two schools for the pilot study, one was randomly assigned as the control group and the other as the experimental group using a flip of coin with the rule that when a head appeared, the first handpicked bolus went for the experimental whereas and when a tail appeared the first handpicked bolus went for the control. The same procedure was adopted in the selection of experimental and control schools for the main study. This was to reduce bias. Furtherance to the emergence of experimental and control schools for the study, trips were made to the selected schools and their (principals, further mathematics teachers and students) cooperation solicited for the smooth conduct of the study. In all, 96 students participated in the present study. This consisted of 42 in the experimental group and 54 in the control group.

Research instrument

The data needed in this study were gathered using one research instrument tagged Beliefs about Further Mathematics Questionnaire (BFMQ) before and after intervention conditions enacted by the researcher and the participating teachers.

Beliefs about further mathematics questionnaire (BFMQ)

The Beliefs about Further Mathematics Questionnaire consisted of 28 four-point scale items, anchored on Strongly Agree, Agree, Disagree or Strongly Disagree, to which the students were asked to respond. The BFMQ was purposely used in this study as pre- and post- test in both the experimental and control classes and considered appropriate as a questionnaire for this study because of its ‘‘versatility, efficiency and generalisabilty’’ (McMillan, 2000). The versatility of a questionnaire lies in its ability to address a wide range of problems or questions, especially when the purpose is to describe the beliefs, attitudes and perspectives of the respondents. Its limitation, according to Mertler & Charles (2005), is that it does not allow the researcher to probe further as would be possible in an interview. The BFMQ was developed by modifying the 18-items on beliefs about mathematics survey developed by Perry et al. (2002); and then adding ten other beliefs items constructed by the researcher to make 28 items. The survey items by Perry et al (2002) which had been widely used in previous researches in Australia (Perry et al., 2005; Perry, Howard & Tracey, 1999; Perry, Howard & Conroy, 1996) were modified by replacing mathematics with further mathematics and based on experts' advice 10 other beliefs items in relation to the nature of Further Mathematics, its teaching and the theoretical underpinning of the Further Mathematics curriculum were constructed (Harbour-Peters, 1990, 1991). The original survey items by Perry et al (2002) were considered appropriate and suitable but inadequate by two experts in mathematics education in a tertiary institution in Nigeria. The 10 items were also scrutinised by the two mathematics educators and minor amendments were made. The suitability of the newly developed BFMQ rested on the fact that it enabled the researcher to examine the impact of PBL approach on students' beliefs about further mathematics. This is one of the aims of the study. One advantage of the BFMQ was that it provided an overview of commonly espoused students' beliefs since it was based on statements summarizing modern approaches to further mathematics learning and teaching.

The BFMQ scores of the study sample were subjected to exploratory factor analysis using Principal Components Analysis with the factor loadings ranging from .402 to .818 based on an Oblimin three factor resolution.

Prior to this, the data screening process on the responses of the participants showed no missing values and no concern about normality, linearity, multicolinearity, and singularity. For example, subscale scores were normally distributed with skewness and kurtosis values within acceptable ranges (e.g. skewness ranged from -1.48 to 1.36, kurtosis ranged from -3.85 to 3.03) as Kline (1998) suggested using absolute cut-off values of 3.0 for skewness and 8.0 for kurtosis. Also, inspection of the correlation matrix of the 28 items revealed that the correlations when taken overall were statistically significant as indicated by the Bartlett's test of sphericity, χ^2 = *2228*.779*; df=378; p*<.001 which tests the null hypothesis that the correlation matrix is an identity matrix. The Kaiser-Meyer-Olkin measure of sampling adequacy (MSA) fell within acceptable range (values of .60 and above) with a value of .836. Each of the variables also exceeded the threshold value (.60) of MSA which ranged from .616 to .892. Finally, most of the partial correlations were small as indicated by the anti-image correlation matrix. These measures all led to the conclusion that the set of 24 items of beliefs about further mathematics was appropriate for PCA.

In running the factor analysis, the researchers observed the following criteria for determining the number of factors. First, consideration was given to the option of retaining those factors whose meaning is comprehensible. Second, the Kaiser (1960) rule, which suggests five factors and ascertains that all components with eigenvalues under 1.0 be dropped, was observed. The method is not recommended when used as the sole cut-off criterion for estimating the number of factors as it tends to over extract factors (Gorsuch, 1983). Third, the variance explained criterion was observed. This involves keeping enough factors to account for 90% (sometimes 80%) of variation, and where the goal of parsimony is emphasised the criterion could be as low as 50%. Fourth, scree test, which suggests three factors, was plotted. The Cattell scree test plots the components as the X-axis and the corresponding eigenvalues as the Y-axis. As one moves to the right, toward later components, the eigenvalues drop. When the drop ceases and the curve makes an elbow toward less steep decline, Cattell's scree test says to drop all further components after the one starting the elbow (Gorsuch, 1983). This has been criticized for being amenable to researcher-controlled fudging. That is, picking the elbow can be subjective. In this study, a five- factor solution was initially obtained. This was considered not good enough as one of the components had just two items.

A four-factor solution was thus computed but this was also jettison because one of the factors with only three items had low internal consistency reliability (.23). However, an examination of the scree plot of eigenvalues gave an indication suggestive of three-factor solution. The three-factor solution was thus computed and this was found not only meaningful but had non-overlapping interpretable structures. That is, items did not load on more than one structure. Fig. 1 below further confirmed the three- factor solution.

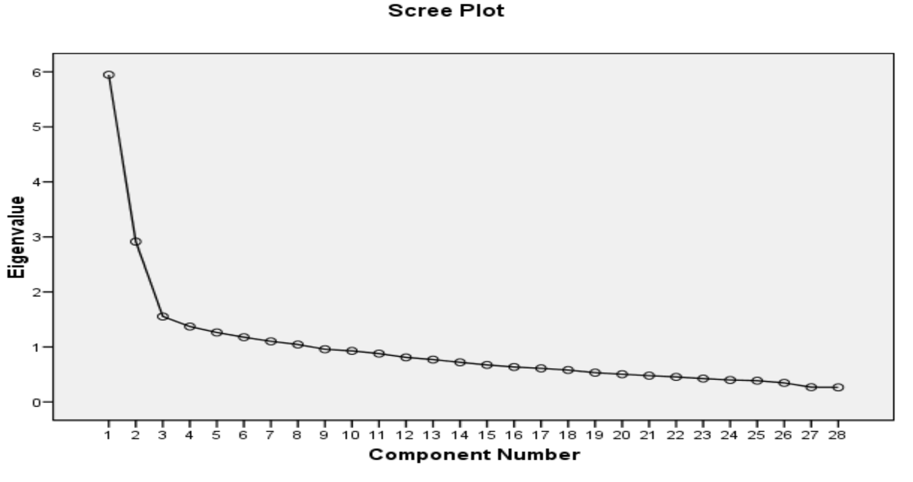

Fig. 1. Cattel scree plot showing number of components and eigenvalues of the correlation matrix

There are two rotation methods in factor analysis namely orthogonal and oblique (Bartholomew et al., 2008). Varimax rotation is an orthogonal rotation of the factor axes to maximize the variance of the squared loadings of a factor (column) on all the variables (rows) in a factor matrix, which has the effect of differentiating the original variables by extracted factor. A varimax solution yields results that make it easy to identify each variable with a single factor and it is the most common rotation option. The direct oblimin rotation is the standard method when one wishes a non-orthogonal (oblique solution) - that is, one in which the factors are allowed to be correlated. This will result in higher eigenvalues but diminished interpretability of the factors. However, the researchers wished a non-orthogonal solution and so, adopted the direct oblimin rotation. Factor 1 is composed of 15 items (5, 6, 7, 8, 12, 13, 15, 16, 18, 21, 22, 23, 24, 25, and 26) reflecting students' cognitive beliefs about the teaching and learning of Further Mathematics and accounted for 21.24% of the item variance. Factor 2 contained seven items (1, 2, 4, 10, 17, 20, and 27) and reflected students' beliefs about the nature and importance of Further Mathematics and accounted for 10.41% of the item variance. Factor 3 is made up of six items (3, 9, 11, 14, 19, and 28) and showed students' beliefs about aesthetic value and teachers' behaviour in Further Mathematics and accounted for 5.55% of the item variance. The three interpretable factors accounted for 37.19% of the item variance. The three identified factors are clearly different and non-overlapping. This indicates that it is possible for a student to hold both beliefs simultaneously. Cronbach alpha computed to determine the internal consistency and reliability of the BFMQ was 0.86. The internal consistency reliabilities of factors 1, 2, and 3 were 0.76, 0.84, and 0.81 respectively.

Procedure for data collection

The study covered a period of three months. Prior to the commencement of teaching in the experimental and control classes, students were pretested on the BFMQ. The essence of the pre-treatment was to ascertain the background knowledge of the participants in both the experimental and control classes before entering into the experiment/instruction. The attention of the regular mathematics graduate teacher in the control school was sought after the management of the school had given approval for the study to be conducted in the school. The details of the study were neither made known to him nor fully discussed with the school management as the study was presented to the duo as if the exercise was meant for the school alone. This was to prevent any form of bias and influence on the part of the teacher in the course of his teaching.

The participating teacher in the control school unlike his counterparts at the experimental school was not trained on the PBL approach but the researchers paid unscheduled visits to the control school during the school hours and this afforded the researchers the opportunity to observe the teacher while teaching. However, no attempt was made to discuss the classroom interaction pattern that prevailed between the teacher and the students in the classroom. He taught the students with the traditional method following the already prepared instructional plan within the context of the contents selected for the study. The teacher covered the topics related to the Indices and Logarithms, Algebraic Equations, and Series and Sequences. The instructional lesson plan in the control school differed only from that of the experimental school in the area of presentation. The presentation in the control school followed the routine traditional activities against the flowchart of problem solving process enacted in the experimental school. The traditional mathematics instruction involved lessons with lecture and questioning methods to teach the concepts related to indices and logarithms, algebraic equations, and series and sequences. The students studied the approved mathematics textbooks on their own before the class hour. The teacher structured the entire class as a unit, wrote notes on the chalkboard about definitions of concepts related to indices and logarithms, algebraic equations and sequences and series. The teacher worked examples on the chalkboard about indices and logarithms, algebraic equations and sequences and series, and, after his explanation, students discussed the concepts and examples with teacher-directed questions. For the majority of instructional time in the control school, students received instruction and engaged in discussions stemming from the teacher's explanations and questions. Thus, teaching in the control school was largely teacher-dominated and learning confined to the classroom. The classroom instruction in the control class was two periods of 40 minutes each per week in the afternoon on Tuesdays and

Thursdays. The afternoon periods on these two days were uniform across the schools offering FM in the local government area of the study.

The researchers sought the consent of the management of the experimental school and an approval was given to conduct the study in the school. The nature and purpose of the research were then explained to the four teachers who showed willingness and readiness to participate in the study. The highlight of the weekly activities that would be carried out and the extent of their involvement were discussed with them. The teachers were given comprehensive orientation on the principle behind the PBL as an instructional strategy and content areas for the study discussed. They were free to ask questions and offer suggestions on how best this modern approach could successfully be implemented in the school. Because the PBL was a novel approach for participating teachers in the experimental group, one of the researchers (first author) taught students in the experimental group in order to ensure fidelity of treatment. The first author acted as both a teacher and a researcher in the experimental class based on the following reasons: Although many teachers are aware of problem solving, few teachers understand the difference between a traditional approach and problem-based approach. For those teachers who understand what problem-based approach entails, majority are neither sure of how to implement this approach in their classrooms nor are they interested in even to try it (due to their own valid reasons).

Prior to the actual implementations of PBL in the experimental classroom, one of the researchers in collaboration with the four participating mathematics graduate teachers grouped the 42 Further Mathematics students heterogeneously based on their performances at the Junior Secondary School (JSS) year 3 final examinations. The class was referred to by the researchers as Learners' Community Group (LCG) that consisted of six groups of seven students each. The sitting arrangement was re-constituted in a semi-circular form that made it possible for teachers to walk across the groups. The groups were coded as LCG A, B, C, R, P, and Q. The students were asked to construct nametags that were used as a form of identification. The students coded numbers were LCGA 01-07, LCGB 01-07, LCGC 01-07, LCGR 01-07, LCGP 01-07 and LCGQ 01-07. The coded number for the students was used for 'blind' assessment.

The seats were arranged for all students to face the chalkboard. Files were provided for all the students with working sheets. Shipboard, cello tape, markers of different colours and exercise books were given to the participating teachers to note their remarks and observations. Two periods of forty minutes each were allocated to the teaching of further mathematics in a week. The periods were usually in the afternoon on Tuesdays and Thursdays as dictated by the government policy. Thus, the researchers had no control on the placement of FM in the afternoon on the school timetable. The rigidity of the timetable did not allow the researchers to create more instructional time in the teaching of the contents in the experimental class and more importantly, the school authority in compliance with the State Government's directives did not allow any extension of classroom activities beyond the closing time. This precluded any intruder in the PBL classroom that could have created an unusual atmosphere.

Four mathematics graduate teachers at the experimental school watched the researcher leading discussions in the further mathematics classroom using PBL in a scaffolding manner to suit the already prepared instructional lesson plan. The instructional plan consisted of Introduction, Objectives, Content, Presentation, Evaluation and Conclusion. In the experimental class, the PBL group process adopted consisted of five phases namely: (i) identify the problem; (ii) make assumptions; (iii) formulate a model; (iv) use the model; and (v) evaluate the model. Aside the arrangement of students into heterogeneous ability groups, the flowchart on problem processes for PBL used in this study consisted of five phases: Identify the problem, Make assumption, Formulate a model, Use the model, and Evaluate the model as seen in Fig. 2 below.

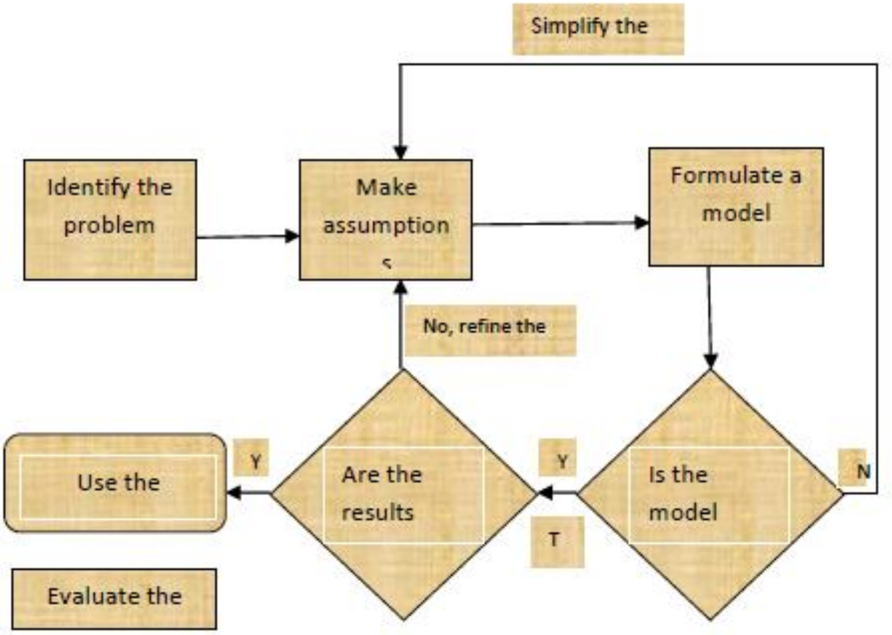

Fig. 2. The PBL group process

In the first contact period of the third week in the PBL class, students were given orientation on the PBL and its associated problem-solving processes. This was followed by a diagnostic test on indices in which students were to investigate the correctness of the given equations: (i) $2^2 \times 3^3 = 6^6$? (ii) $(2^3)^4 = 2^7; 6^4; 2^{12}; 16^3$? (Pick the correct answers). Students were left to ruminate on the given tasks individually and in groups following the identified problem-solving processes while the teacher acted as a facilitator. One member each from the first three groups (LCG A, B & C) was selected by the teacher to make presentations on the chalkboard while other members of the learners' community group critiqued the presentations and this triggered off dialogue in the classroom. Thus, mixed feelings ensued among members of the learners' community group as some were in favour that the equality holds for the first equation, some were against this stand and obtained 6^5 as the solution while others were indifferent. In reaching consensus among the three opposing groups, the teacher interjected by calling the students attention to simplify the value on the right hand side of the equation and see whether it corresponds to the simplified value on the left hand side. This made the three opposing groups to recline on their decisions and agreed that the equality did not hold and stemming from the teacher's questions, a member of the class stated that the law of indices could not be applied to the given equation because the given numbers were not of the same base.

The entire class was in agreement with the final submission while another member of the class gave a brisk overview of the laws of indices. In the second given equation, students engaged in individual and group investigations of the task following the identified problem-solving processes and the same procedure as described above took place in arriving at final answers while the teacher acted as a facilitator. Similar procedure was adopted in teaching topics related to the logarithms in the fourth week, algebraic equations in the fifth and sixth weeks and sequences and series in the seventh, eighth and ninth weeks of the study. In each of the topics taught students were given ill-structured task as homework that demanded their visiting the libraries, and surfing the net in preparation for presentation in the next contact period.

Data analysis

The quantitative data collected using BFMQ were analysed using the measures of central tendency of means and standard deviations, which are important precursor to conducting inferential statistical analysis of t-test. This study tested differences in students' beliefs about further mathematics before and after treatment conditions in both the experimental and control classes and no attempt was made to test relationships. Thus, this foreclosed the adoption of correlation statistic. The t-test statistic was adopted in the study partly because two groups were involved and more importantly, the statistic is considered more robust when comparing differences of two means. Independent Samples t-test was used to analyse the pre-test and post- treatment scores of the control and experimental groups for BFMQ. Analysis of variance (ANOVA) was also considered appropriate in this study to test the null hypothesis and since it generalizes the t-test value. Thus, a one-way ANOVA was adopted to corroborate results obtained using the t-test and also to prove the relation $F = t^2$. An alpha level of 0.05 was used for all statistical tests.

Results

Research question one

What are the beliefs of senior secondary school students towards further mathematics teaching and learning?

Mean and standard deviation of the scores acquired from BFMQ in relation to research question one by further mathematics students were presented in Table 1 below. When the beliefs about further mathematics scores of the participants were analysed, it was revealed that their beliefs about further mathematics was high with the mean of 3.17 (SD=.42).

Table 1. Results of statistical analysis of post-treatment scores on BFMQ

	Control class (n = 54)			**Experimental class (n = 42)**		
Beliefs Statements	**Mean ($\bar{x}$)**	**SD**	**Rank**	**Mean ($\bar{x}$)**	**SD**	**Rank**
Theme 1: cognitive beliefs about the teaching and learning of Further Mathematics						
5: Right answers are much more important in further mathematics than the ways in which you	**3.80**	**.63**	**2**	**3.45**	**.74**	**9.5**

get them						
6: Further mathematics knowledge is the result of the learner interpreting and organizing the information gained from experiences	**2.85**	**1.17**	**13**	**3.48**	**.83**	**8**
7: Being able to build on other students' ideas make extensions of FM real	**1.83**	**1.04**	**15**	**2.90**	**.98**	**15**
8: Students are rational decision makers capable of determining for themselves what is right and wrong	**3.17**	**1.11**	**8**	**3.76**	**.43**	**2.5**
12: Students should be allowed to use any method known to them in solving FM problems	**3.07**	**1.06**	**10.5**	**3.52**	**.83**	**7**
13: Young students are capable of much higher levels of mathematical thought than has been suggested traditionally	**3.07**	**1.08**	**10.5**	**3.45**	**.80**	**9.5**
15: Being able to memorize facts is critical in FM learning	**2.96**	**1.24**	**12**	**3.26**	**1.04**	**13**
16: Further mathematics learning is enhanced by activities which build upon and respect students' experiences	**3.15**	**1.05**	**9**	**3.40**	**.77**	**11**
18: Teachers should provide instructional activities which result in problematic situations for learners	**2.22**	**1.16**	**14**	**3.29**	**.77**	**12**
21: The role of the FM teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge	**3.48**	**.93**	**6**	**3.12**	**.97**	**14**
22: Teachers should recognize that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding	**3.56**	**.97**	**5**	**3.76**	**.58**	**2.5**
23: Teachers should negotiate social norms with the students	**3.57**	**.92**	**4**	**3.74**	**.50**	**4**

in order to develop a cooperative learning environment in which students can construct their knowledge						
24: Further mathematics concepts enable students to interpret and solve applied problems	**3.59**	**.92**	**3**	**3.71**	**.51**	**5**
25: Further mathematics is a product of the invention of human mind	**3.89**	**.46**	**1**	**3.81**	**.46**	**1**
26: Further mathematics is abstract	**3.46**	**.82**	**7**	**3.64**	**.58**	**6**
Sub-overall	**3.18**	**.97**		**3.49**	**.72**	
Theme 2: Beliefs about the nature and importance of Further Mathematics						
1: Further mathematics is computation	**1.85**	**1.25**	**7**	**3.55**	**.83**	**2**
2: Further mathematics problems given to students should be quickly solvable in a few steps	**2.33**	**1.30**	**4**	**3.67**	**.65**	**1**
4: Further mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking	**2.43**	**1.28**	**3**	**3.43**	**.74**	**5**
10: Periods of uncertainty, conflict, confusion, surprise are a significant part of the FM learning process	**2.13**	**1.20**	**6**	**3.50**	**.80**	**3.5**
17: Further mathematics learning is enhanced by challenge within a supportive environment	**2.28**	**1.17**	**5**	**2.95**	**.91**	**7**
20: Teachers or the textbook – not the student – are authorities for what is right or wrong	**3.41**	**.98**	**1**	**3.19**	**.92**	**6**
27: Further mathematics is the bedrock of Science and Technology	**2.89**	**1.09**	**2**	**3.50**	**.77**	**3.5**
Sub-overall	**2.00**	**1.18**		**3.40**	**.80**	
Theme 3: Beliefs about aesthetic value and teachers' behaviour in Further Mathematics						

3: Further mathematics is the dynamic searching for order and pattern in the learner's environment	**2.30**	**1.30**	**6**	**3.36**	**.66**	**5**
9: Further mathematics learning is being able to get the right answers quickly	**3.43**	**.94**	**1**	**3.38**	**.96**	**4**
11: Further mathematics teachers make learning more meaningful to students when problems are taken from real-life context	**2.98**	**1.11**	**3**	**3.55**	**.74**	**2.5**
14: Teachers' should not rebuke students' for not answering questions correctly	**2.70**	**1.16**	**5**	**3.67**	**.79**	**1**
19: Teachers should encourage students to ask why they have to learn some FM topics	**3.06**	**1.14**	**2**	**2.79**	**1.05**	**6**
28: Teachers' should encourage students to formulate solution procedures by themselves in trying to solve real-world problems	**2.76**	**1.23**	**4**	**3.55**	**.71**	**2.5**
Sub-overall	**2.87**	**1.15**		**3.38**	**.82**	
Grand-overall	**2.89**	**.48**		**3.44**	**.36**	

Research question two

Will there be any significant difference in the post-treatment scores on Beliefs about Further mathematics questionnaire (BFMQ) between students exposed to the PBL and those exposed to the TM?

Table 2 below shows that the post-treatment BFMQ mean score for the experimental class *(M=3.44)* was higher than the mean score of the control class *(M=2.89)*, an indication that the experimental students had stronger beliefs about Further Mathematics when compared with their counterparts in the control class. The standard deviation of the post-treatment BFMQ scores for the experimental class (*S.D =.36)* was lower than the standard deviation of the control class (*S.D=.48),* an attestation that scores obtained by students in the experimental class clustered around the mean while scores obtained by control class were spread away from the mean. The mean gain (.88) in the experimental class was above the mean gain (.06) recorded in the control class. The mean difference of 0.55 between the experimental and control classes after treatment was significant *(t=-6.22, p=.000)* as indicated by the independent samples t-test results in Table 2 below. The significant result at a level of $p<0.05$ meant that there was a less than 5% chance that the result was just due to randomness. The flip side of this was that there was a 95% chance that the difference in post-treatment score on BFMQ between the experimental and control classes was a real difference and not just due to chance. As observed in the table below, the two-tailed p value was 0.000 meaning that random sampling from identical populations would lead to a difference smaller than was observed in 100% of experiments and larger than was observed in 0% of experiments (study carried out). Thus, there was a significant difference in the post-treatment scores on BFMQ of students between the experimental and control classes.

Table 2. Means, standard deviations, and t-test values on pre-and post-treatment scores on BFMQ for Experimental and Control classes

Test Occasion		Experimental Class	Control Class	t	p
Pre	*M*	2.57	2.74		
	SD	.45	.37	2.13*	.036
	N	42	54		
Post	*M*	3.44	2.89		
	SD	.36	.48	-6.22*	.000
	N	42	54		

*significant at p<.05 level

Null hypothesis one

There is no statistically significant difference between the post-treatment scores on BFMQ of students exposed to the PBL and those exposed to the TM.

Further analysis of the post-treatment scores on BFMQ of students in both the experimental and control classes using one-way ANOVA as contained in Table 3 below showed that difference in means between the two classes was significant ($F_{(1,95)} = 38.49$; $p = .000$).

Table 3. One-way ANOVA on post-treatment score on BFMQ for Experimental and Control classes

	Sum of squares	**Df**	**Mean Square**	**F**	**Sig.**
Between groups	7.204	1	7.204	38.49	.000
Within groups	17.595	94	.187		
Total	24.800	95			

Since the ANOVA generalises the t-test to more than two groups, it is apparent that the relation $F = t^2$ must hold when $t = -6.20$. However, the p value of 0.000 recorded on the ANOVA table above tallied with the p value obtained in the t-test. Thus, research hypothesis one was rejected. Hence, there was a statistically significant difference between the post-treatment scores on BFMQ of students exposed to the PBL and those exposed to the TM.

Discussion

The present study showed that the overall belief about further mathematics scores of the participants was high (M=3.17, SD=.42) and this indicates that the participants held strong beliefs about further mathematics teaching and learning. In assessing beliefs, Giovanni & Sangcap (2010) carried out a study that aimed at analyzing possible significant differences in mathematics related beliefs, related to gender, year level and field of specialization. The results of the study showed positive beliefs that Filipino students valued effort in increasing ones mathematical ability and considered mathematics as useful in their daily lives.

The present study also revealed that the mean pre-treatment score on BFMQ of the students in the experimental group was statistically significantly different from that of the students in the control group in favour of the latter. This is an indication that the two groups showed remarkable difference in their responses to the beliefs about further mathematics questionnaire prior to the intervention. Thus, the two groups did not enter the instruction/experiment on equal footing and any observable significant difference in the mean post-treatment score on BFMQ of the two groups could be attributed to chance. Going by the results of data analysis presented in the preceding section for research hypothesis two and null hypothesis one there was a significant difference in post-treatment scores on beliefs about further mathematics between students exposed to the PBL and those taught with the TM. This finding revealed that students treated with the PBL recorded stronger beliefs about further mathematics than their counterparts exposed to the traditional instruction. Although, literature is scanty on the relation between PBL and students' beliefs, evidence suggests that PBL has no positive impact on students' beliefs (Sahin, 2009a) and this ran contrary to the findings of the present study. Şahin (2009b) found that PBL and traditional groups displayed similar degree of 'expert' beliefs. He maintained that the results of his study showed that university students' expectations and beliefs about physics and physics learning have deteriorated because of one semester of instruction whether in PBL or traditional context. Cotič & Zuljan (2009) found no significant effect of PBL on students' attitude toward mathematics.

This difference in mean score in favour of the experimental class in the present study showed the efficacy of the use of PBL in promoting students' beliefs about further mathematics thereby supporting previous research findings that indicated that the PBL is an effective strategy for stimulating students' learning outcomes (Williams et al., 1998; Sungur & Tekkaya, 2006; Iroegbu, 1998; Wheijin, 2005; Gordon et al., 2001; Gallagher & Stepien 1996) like other learner-centred instructional strategies (Awofala et al., 2013; Awofala, 2011; Awofala et al., 2012). The effectiveness of the PBL on students' beliefs about further mathematics recorded in this study coincided with previous research findings on self-regulated learning. Sungur & Tekkaya (2006) found that the PBL students had higher levels of intrinsic goal orientation, task value, use of elaboration learning strategies, critical thinking, metacognitive self-regulation, effort regulation, and peer learning compared with control group students treated with the traditional instruction. Similarly, Gordon et al. (2001) found that the PBL students value the student-centred nature of PBL, including information seeking, high levels of challenge, group work, and personal relevance of the material. Although these researchers did not consider beliefs as a dependent variable but beliefs and self-regulated learning fall under the same domain called affective and each has been found to impact students' achievement (Andreassen & Rees, 2005; Furinghetti & Pehkonen, 2000). The noticeable impact of PBL on students' beliefs about further mathematics recorded in this study may be attributed to the features inherent in the use of PBL. PBL offers students opportunity to analyse and discuss problems so that they can realise gaps in their knowledge base, determine their strengths and weaknesses, control their own learning, and develop self-regulatory skills (Glaser as cited in Karabulut, 2002). The learning outcomes of students in mathematics are strongly related to their beliefs and attitudes towards the subject (Andreassen & Rees, 2005; Furinghetti & Pehkonen, 2000; Leder et. Al., 2002; Pehkonen, 2004; Schoenfeld, 1992, Thompson, 1992) and as suggested by previous studies (Pekhonen, 2004; Mason, 2003, Kloosterman & Stage, 1992); the existence of a system of beliefs affects students' behaviour which may either impede or facilitate understanding when students solve mathematical problems. According to Karabulut (2002) PBL creates an environment in which students actively participate in the learning process, take responsibility for their own learning, and become better learners in terms of time management skills and ability to identify learning issues and to access resources. In this study, PBL not only allowed the arrangement of students into heterogeneous ability groups but also facilitated students' adoption of problem solving process of identifying the problem, making assumptions, formulating a model, using the model and evaluating the model within the teachers' scaffolding role. Scaffolds are forms of support provided by the teacher (or another student) to help students' bridge the gap between their current abilities and the intended goals. Observations in the PBL classroom revealed that students thought to be shy and passive during Further Mathematics lessons suddenly became active participants following PBL instruction thereby making the perceived low able students rank shoulder to shoulder with the brilliant ones in the further mathematics lesson. Thus, it is concluded that students exposed to the PBL held stronger beliefs about further mathematics than their counterparts that were treated with the traditional method.

Conclusion

This study has shown the effectiveness of PBL in enhancing students' beliefs about further mathematics. However, one major limitation of this study is that the study relied on the purposive sampling technique in choosing schools that participated in the study. This was due to few in number of students taking further mathematics consequent upon paucity of qualified graduate mathematics teachers in the study area in particular and generally in Nigeria. This non-probability sampling is often criticised for being subjective to researcher's manipulation, thus making generalisation of findings impractical. This is seen as a potential weakness of the study. The ability to address a wide range of problems or questions, especially when the purpose is to describe the beliefs, attitudes and perspectives of the respondents is one of the strengths of a questionnaire. It does not however allow the researcher to probe further as would have been possible in an interview (Mertler & Charles, 2005). This is seen as another potential weakness of the study. Future study may look at the effect of PBL on each of the dimensions of beliefs about further mathematics and this may lend itself to the use of more robust statistical tool of multivariate analysis of variance. However, it was recommended that efforts should be made to integrate the philosophy of PBL into the preservice teachers' curriculum at the teacher-preparation institutions in Nigeria.

NOTES

1. http://timssandpirls.bc.edu/timss1995.html

2. http://www.urban.org/toolkit/data-methods/quasi-experimental.cfm?renderforprint=1

3. *Population and housing census of the Federal Republic of Nigeria: national and state population and housing*. National Population Commission, 2006.

4. *National Curriculum for Further-Mathematics*. Federal Ministry of Education. Lagos: NERDC Press, 1985.

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