AN EXPLORATORY STUDY ON NUMBER OF CHILDREN DESIRED AND WHAT ACCOMPLISHED IN FAMILY SETTINGS

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Abstract. In any family settings, racing children is a big decision that requires serious self-reflecting and communication between couples. In African settings, there is usually a rift in the agreement of the number and the gender of children to be borne by couples; while the man prefers a male child, the wife may prefer a female child instead. The number of children by the couple also determine the kinds of education those children will eventually have later. To this effect, in this research work, we want to study the Man's proposed and actual number of children; the degree of association in the man's decision using Quasi symmetry and Homogeneous Agreement model; how well some factors (Age, Religion, Family status, Occupation, Level of education and Ethnic group) influence the number of children; and to know the stopping rule for child bearing by the man. It was observed that 16.2% of the respondents had above the number of children proposed when they stopped bearing children, 21.5% of the respondents had below the number of children proposed when they stopped while 62.3% of the respondents had the exact number of children proposed when they eventually stopped bearing children. We observed that Age and Religion influence the number of children. We also observed that the probability (p) of having at least one male child is 0.8019 based on the available data. The chance of any newly wedded couple ever having a male child at any trial follows a geometric distribution $f(x) = (0.8019)(0.1981)^{x-1}, x = 1,2,3\ldots$ Quasi symmetry model has a better fit for agreement measure than Homogeneous agreement model.

Keywords: agreement, association, child bearing, family settings, quasi symmetry, homogeneous agreement, geometric distribution

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Introduction

Racing children is a big decision that requires couples to do some serious self-reflecting and communication. However, some couples do not exactly contemplate parenthood or they have wrong idea about racing children. Some mistakenly assumed that having a child will fix their relationship problems and bring them closer. Unfortunately, this usually backfires, because the new stressors that come with having baby just amplify existing issues.

Other couples decided to have kids because they think it's simply the next step after matrimony. Many couples do not give themselves permission to thoughtfully explore whether or not having children is right for them because of fear of being different disappointing others or missing out on life experiences that couples with children experience. Relationship satisfaction also is critical. A couple needs to have a healthy satisfying relationship with a clear understanding of, and strategies for working with the pitfalls in their relationship.

The number of children actually born to a couple is determined by the capacity to bear children, the factors that determine desired family size, and couple's ability to achieve its aims. The number of children that a couple desires is also the outcome of complex calculations.

This study is designed to measure how true the proposed and actual number of children by the man is being validated using Quasi-symmetry and Homogeneous Agreement model. Specific objectives are to: compare the Man's proposed and actual number of children; measure the degree of association in the Man's decision using Quasi symmetry and Homogeneous Agreement model; measure how well some factors (age, religion, family status, occupation, level of education and ethnic group) influence the number of children; and construct a probability model for a newly wedded couple ever having a male child.

The data for this study is a primary data in which questionnaires were designed to collect information from the head of the family (Man). Section A of the questionnaire discussed the demographical variables; Section B discussed the proposed and actual numbers of children while the last Section discussed factors influencing their decision on the proposed and actual number of children. A total of 500 questionnaires were administered and 303 questionnaires were harvested.

Methodology

For a given $I \times I$ contingency table, let π_{ij}be the probability of cell i, j. Also let R_1 and R_2 be row and column labels, respectively. There exist Symmetry if

$$\pi_{ij} = \pi_{ji}, whenever\ i \neq j.$$

Let m_{ij} be the expected value of the cell i, j, such that

$$m_{ij} = n\pi_{ij}$$

Then symmetry model as log-linear model is

$$\log(m_{ij}) = \mu + \lambda_i + \lambda_j + \lambda_{ij}, \quad i, j = 1,2, \dots, I \tag{1}$$

where $\lambda_{ij} = \lambda_{ji}, \sum_{i=I}^{I} \lambda_i = 0, and\ \sum_{i=\mathrm{I}}^{I} \lambda_{ij} = 0\ for\ j = 1,2, \dots, I$. There are no superscripts on the main or marginal effect terms because they are the same for rows and columns, that is, $\lambda_i^{R_1R_2} = \lambda_j^{R_1R_2}$ when $i = j$. In other words, the row and column margins are equal, that is $m_{i+} = m_{+i}$ (Tanner & Young, 1985; Adejumo, 2005).

For general loglinear model with Poisson as the underlying sampling distribution, the log-likelihood equation is given as

$$l\left(p(m_{ij})\right) = logL\left(p(m_{ij})\right)$$

$$= \sum_{ij} n_{ij} \log(m_{ij}) - \sum_{ij} m_{ij} + \left[\sum_{ij} \log(n_{ij}!)\right]^{-1} \qquad (2)$$

By incorporating symmetry model into Eq. 2, the likelihood equations are

$$\hat{m}_{ij} + \hat{m}_{ji} = n_{ij} + n_{ji}\ for\ all\ i\ and\ j.$$

The Kernel of the log-likelihood is

$$\sum_{ij} n_{ij} \log(m_{ij})$$

$$= n_{++}\mu \qquad (3)$$

$$+ \sum_{i} (n_{i+} + n_{+i})\lambda_i + \sum_{ij} \left\{\frac{n_{ij} + n_{ji}}{2}\right\} \lambda_{ij}$$

Maximizing this equation yields the following expected cell values

$$\widehat{m}_{ij} = \begin{cases} \dfrac{n_{ij} + n_{ji}}{2} & if\ i \neq j, \\ n_{ii} & if\ i = j, \end{cases} \qquad (4)$$

The goodness of fit statistics, Pearson's chi-square statistic χ^2 as well as the likelihood ratio statistic G^2 shall be used to test the models (Yule, 1912; Wilks, 1935).

The degrees of freedom for the residual (df) is obtained as (number of cells) minus (number of non-redundant parameters) which is mathematically given as $\frac{I(I-1)}{2}$.

The two statistics have asymptotic χ^2 distribution with the above degrees of freedom under the null hypothesis that the symmetry model fits.

Quasi symmetry model (QS)

Quasi-symmetry model was introduced by Caussinus (1965) as an extension of symmetry model. There are a number of equivalent definitions, one given by McCullagh (1978) is

$$\pi_{ij} = c\frac{\alpha_i}{\alpha_j}\emptyset_{ij} \qquad (5)$$

with $\emptyset_{ij} = \emptyset_{ji}, \sum\sum\emptyset_{ij} = 1$, $\alpha_i = 1$ and c a constant to make$\sum\sum\pi_{ij} = 1$. Quasi-symmetry model according to McCullagh (1978) is permutation invariant, such that if an arbitrary permutation is applied to both rows and columns, the new cell probability $\pi_{ij}^{'}$ are given by

$$\pi'_{ij} = c\frac{\propto'_i}{\propto'_j}\emptyset'_{ij} \qquad (6)$$

Where $\propto'$ is a permutation of the elements of α and $\emptyset'$ is obtained from $\emptyset$ by permuting both rows and columns.

The loglinear form of Quasi-symmetry model is given as

$$\log(m_{ij}) = \mu + \lambda_j^{R_2} + \lambda_{ij}^{R_1R_2} + \lambda_{ij}^{R_1R_2} \qquad (7)$$

where $\lambda_i^{R_1} \neq \lambda_i^{R_2}$ and $\lambda_{ij}^{R_1R_2} = \lambda_{ji}^{R_1R_2}$ for $i \neq j$.

QS can also be written as

$$\log(m_{ij}) = \mu + \lambda_i + \lambda_j + \alpha_j + \lambda_{ij} \qquad (8)$$

where $\sum_i \lambda_i = 0, \sum_j \lambda_j = 0, \sum_i \alpha_i = 0, and$ $\sum_j \lambda_{ij} = 0$ for $i = j = 1,2, \dots, I$. This model is a special case to symmetry when $\alpha_j = 0$ for all j.

This also treats the classification as nominal but it does not imply Marginal Homogeneity. This multiplicative form of QS model is

$$\pi_{ij} = \alpha_i \beta_j \tau_{ij} \qquad (9)$$

The likelihood equations for the model are

$$\widehat{m}_{i+} = n_{i+}$$

$$\widehat{m}_{+j} = n_{+j}$$

$$\widehat{m}_{ij} + \widehat{m}_{ji} = n_{ij} + n_{ji}, for\ i \neq j$$

$$\widehat{m}_{ii} = n_{ii}\ for\ i = 1,2, \dots, I$$

Given that $u_1 < u_2 < \cdots < u_I$ and$v_1 < v_2 < \cdots < v_I$, which are the rows and columns scores $u_1$and $v_1$respectively, then the *Ordinal quasi-symmetry model* is given as

$$\log(m_{ij}) = \mu + \lambda_i + \lambda_j + \beta u_j + \lambda_{ij} \qquad (10)$$

Which is a special case to QS model (Eq. 7) for nominal scale data in which

$$\lambda_j^{R_1} - \lambda_j^{R_2} = \beta u_j$$

Eq. (9) indicated that QS is a cell-wise product of table of independence and the table of symmetry. In prospective studies, quasi-symmetry may be a useful model only if the response categories are on a nominal scale (Wilks, 1935; McCullagh, 1982; Agresti, 1988; 1992; 1996; Adejumo, 2005).

Homogeneous agreement model (HA)

Homogeneous agreement model is similar to quasi independence model (QI), but the only difference is that, HA has a uniform agreement parameter $\delta I(i = j)$ for all the categories and not separated as in QI.

Homogeneous agreement model is given as

$$\log(m_{ij}) = \mu + \lambda_i^{R_1} + \lambda_j^{R_2} + \delta I(i = j) \quad (11)$$

For $I(i = j)$ is an indicator function

$$I(i = j) = \begin{cases} 1 \; if \; i = j \\ 0 \; if \; i \neq j \end{cases} \quad (12)$$

and $\sum_i \lambda_i = 0, and \; \sum_j \lambda_j = 0 \; for \; i = j = 1,2, \ldots I$. This model adds to the independence model, the parameter δ for cells along the diagonal. When $\delta > 0$, more agreements regarding outcomes along the diagonal occur than would be expected under independence. That is for any given square table,

$$\delta_{ij} = \begin{cases} \delta & if\ i = j \\ 0 & if\ i \neq j \end{cases} \tag{13}$$

The likelihood equations for HA are

$$\hat{m}_{i+} = n_{i+}$$

$$\hat{m}_{+j} = n_{+j}$$

$$\hat{m}_{ii} = n_{ii}\ for\ i = 1,2,\ldots,I$$

and $\hat{m}_{ij}\ for\ i \neq j$ has to be obtained by iterative method.

The residual degrees of freedom (df) for HA is $(I - 2)$.

HA is a special case to QS model (7) in which $\lambda_{ij}^{R_1R_2} = 0\ when\ i \neq j$ (Agresti, 1988; 1992; 1996; Adejumo, 2005; Adejumo et al., 2007; Ato et al., 2011).

Fitting of quasi symmetry (QS) and homogeneous agreement (HA) models

Model fitting is the most important aspect of modern statistical analysis. Consequently there is a need to obtain the estimates for the parameters in the models. There are many methods of estimating these parameters, but we want to focus on the iterative methods; to be precise. Fisher scoring iterative is considered for the Quasi-symmetry (QS) and Homogeneous Agreement (HA) model based on generalized linear models (GLMs) techniques.

Generalized linear model (GLM) procedure is used to fit these models. Poisson sampling is mostly assumed when fitting GLM to categorical data with $I > 2$. The log likelihood function is

$$l(\theta, \emptyset) = \sum_{i=1}^{n} \Big(y_i b(\theta_i) - c(\theta_i)\big(y_i b(\theta_i) - c(\theta_i) + d(y_i, \emptyset)\big) + d(y_i, \emptyset)\Big) \tag{14}$$

where θ subsumes all the θ_i. It could also be written as a function of β and $\emptyset$ because (given the x_i), β determines all the θ_i. The main approach of maximizing β is by maximizing Eq. (14). The fact that $G(\mu_i) = x_i\beta$ suggests a crude approximation estimate: regress $G(y_i)$ on x_i, perhaps modifying y_i to avoid violating range restrictions (such as taking log (0)), and accounting for the differing variances of the observations (Adejumo, 2005; Adejumo et al., 2007).

Fisher scoring iteration is the widely used technique for maximizing the GLM likelihood over β. The basic step is

$$\beta^{(k+1)} = \beta^k - (E\left(\frac{\partial^2 l}{\partial\beta\partial\beta'}\right))^{-1}\frac{\partial l}{\partial\beta} \tag{15}$$

which can also be written as,

$$\beta^{(k+1)} = \beta^k + (-E\left(l''\left(\beta^{(k)}\right)\right))^{-1}l'\left(\beta^{(k)}\right) \tag{16}$$

where$l$ is the loglikelihood function for the entire sample $y_1, \dots, y_N$ and the expectations are taken with $\beta = \beta^{(k)}$. This is the same as Newton step, except that Hessian of l is replaced by its expectation. Fisher scoring simplifies to

$$\beta^{(k+1)} = (X'WX)^{-1}X^{-1}WZ \tag{17}$$

where W is a diagonal matrix with

$$W_{ii} = (G'(\mu_i)^2 b''(\theta_i))^{-1} \tag{18}$$

and

$$Z_i = (Y_i - \mu_i)G'(\mu_i) + x_i\beta \tag{19}$$

Both equations (18) and (19) use $\beta = \beta^{(k)}$ and then derive values of $\theta_i^{(k)}$ and$\mu_i^{(k)}$. The iteration (17) is known as “iteration reweighted least squares”, or IRLS. The weights W_{ii} have the usual interpretation as reciprocal of variances: $b''(\theta_i)$ is proportional to the variance of Y_i and the $G''(\mu)$ factor in Z_i is squared inW_{ii}. Fisher scoring may also be written as

$$\beta^{(k+1)} = \beta^{(k)} + (X'WX)^{-1}X'WZ^* \tag{20}$$

where

$$Z_i^* = (Y_i - \mu_i)G'(\mu_i).$$

Due to the fact that testing symmetry model is an important preliminary analysis for other analyzes which require symmetric table, some of these models take their baseline model as symmetry model,

$$\log(m_{ij}) = \mu + \lambda_i + \lambda_j + \lambda_{ij}, \quad i, j = 1,2, \dots, I$$

where $\lambda_{ij} = \lambda_{ji}, \sum_{i=1}^{I} \lambda_i = 0, and\ \sum_{i=I}^{I} \lambda_{ij} = 0\ for\ j = 1,2, \dots, I$ and $\lambda_i = \lambda_j$ when $i = j$. We need to describe the structure of variables involved in the modification of this model to obtain their estimates as stated in the model. To this effect, we need to create a variable that takes on a unique value for each diagonal cell and a unique value of each pair of cells.

In the case of quasi-symmetry model where $\lambda_i^{R_1} \neq \lambda_i^{R_2}$, but $\lambda_{ij}^{R_1R_2} = \lambda_{ji}^{R_1R_2}$ for $i \neq j$ the quasi-symmetry model has the variables

$$\lambda = \begin{cases} \lambda_1 & if \;\; i = j = 1 \\ \lambda_2^{R_1} & if \;\; i = 2 \\ \lambda_3^{R_1} & if \;\; i = 3 \\ \lambda_4^{R_1} & if \;\; i = 4 \\ \lambda_5^{R_1} & if \;\; i = 5 \\ \lambda_2^{R_2} & if \;\; j = 2 \\ \lambda_3^{R_2} & if \;\; j = 3 \\ \lambda_4^{R_2} & if \;\; j = 4 \\ \lambda_5^{R_2} & if \;\; j = 5 \\ \lambda_{12} & if\,(i,\, j) = (1,2)\,(2,1) \\ \lambda_{13} & if\,(i,\, j) = (1,3)\,(3,1) \\ \vdots & \\ \lambda_{45} & if\,(i,\, j) = (4,5)\,(5,4) \end{cases}$$

where λ_1 is the intercept. All these variables are treated as nominal variables (Adejumo, et al., 2007).

For Homogeneous Agreement model (HA) which adds to the independence model, the homogeneous indicator variable that represents the cells in which the rater agree, $\delta I(ij)$ is defined as

$$\delta I(i = j) = \begin{cases} \delta \; if \; i = j \\ 0 \; if \; i \neq j \end{cases}$$

and can be represented as a matrix of dummies variables, which is equivalent with the identity matrix.

$$\delta I(i = j) = \delta = \begin{Bmatrix} 1 & 0 & 0 \ldots 0 \\ 0 & 1 & 0 \ldots 0 \\ 0 & 0 & 1 \ldots 0 \\ \vdots & \vdots & \vdots \quad \vdots \\ 0 & 0 & 0 \quad 1 \end{Bmatrix}$$

In this case $\delta I(i=j)$ is uniform for all the diagonal cells (Agresti, 1988; 1996; Adejumo, 2005; Adejumo et al., 2007).

Multinomial logistic regression

Like ordinary regression, logistic regression extends to models with multiple explanatory variables. For instance, the model for $\pi(x) = P(Y=1)$ at values $X = (x_1, \ldots, x_p)$ of p predictor is $logit[\pi(x)] = \alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p$

The alternative formula, directly specifying$\pi(x)$, is

$$\pi(x) = \frac{\exp(\alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p)}{1 + \exp(\alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p)} \tag{21}$$

The parameter β_i refers to the effect of x_i on the log odds that Y=1, controlling the otherx_j (Agresti, 1996; Adejumo, 2002).

The mechanics of ML estimation and model fitting for logistic regression are special cases of the GLM fitting. With *n* subjects, one treats the *n* binary responses as independent. Let $x_i = (x_{i1}, \ldots, x_{ip})$ denote setting i of values of *p* explanatory variables $i = 1, \ldots, N$. When explanatory variables are continuous, a different setting may occur for each subject, in which case N=n. The logistic regression model (21), regarding $\propto$ as a regression parameter with unit coefficient, is

$$\pi(x_i) = \frac{\exp(\sum_{j=1}^{p} \beta_j \, x_{ij})}{1 + \exp(\sum_{j=1}^{p} \beta_j \, x_{ij})} \tag{22}$$

When more than one observation occurs at a fixed x_i value, it is sufficient to record the number of observations n_i and the number of successes. We then let y_i refer to this success count rather than to an individual binary response.

Then $\{Y_1, \dots, Y_N\}$ are independent binomials with $E(Y_i) = n_i\pi(x_i)$, where $n_1 + \cdots + n_N = n$. Their joint probability mass function is proportional to the product of N binomial functions,

$$\prod_{i=1}^{N} \pi\,(x_i)^{y_i}[1-\pi(x_i)]^{n_1-y_i}$$

$$= \left\{\prod_{i=1}^{N} exp\left[\log(\frac{\pi(x_i)}{1-\pi(x_i)})^{y_i}\right]\right\}\left\{\prod_{i=1}^{N}[1-\pi(x_i)]^{n_i}\right\}$$

$$= \left\{exp\left[\sum_{\mathrm{i}} \mathrm{y_i} \log\frac{\pi(x_i)}{1-\pi(x_i)}\right]\right\}\left\{\prod_{i=1}^{N}[1-\pi(x_i)]^{n_i}\right\}$$

For model (Eq. 22), the ith logit is $\sum_j \beta_j x_{ij}$, so the exponential term in the last expression equals $exp\left[\sum_i y_i\,(\sum_j \beta_j x_{ij})\right] = exp\left[\sum_j(\sum_i y_i x_{ij})\beta_j\right]$. Also, since$[1-\pi(x_i)] = [1+\exp(\sum_j \beta_j\, x_{ij})]^{-1}$, the log likelihood equals

$$L(\beta) = \sum_i \left(y_i x_{ij}\right)\beta_j - \sum_i n_i log\left[1 + exp\left(\sum_j \beta_j x_{ij}\right)\right] \tag{23}$$

This depends on the binomial counts only through the sufficient statistics $\{\sum_i y_i x_{ij}, j = 1, \dots, p\}$.

The likelihood equations result from setting $\frac{\delta L(\beta)}{\delta \beta} = 0$. Since

$$\frac{\delta L(\beta)}{\delta \beta_j} = \sum_i y_i x_{ij} - \sum_i n_i x_{ij} \frac{exp(\sum_k \beta_k x_{ik})}{1 + exp(\sum_k \beta_k x_{ik})},$$

the likelihood equations are

$$\sum_i y_i x_{ij} - \sum_i n_i \hat{\pi}_i x_{ij} = 0, \ \ j = 1, \dots, p \tag{24}$$

where $\hat{\pi}_i = exp\left(\sum_k \hat{\beta}_k x_{ik}\right)/\left[1 + exp\left(\sum_k \hat{\beta}_k x_{ik}\right)\right]$ is the ML estimate of $\pi(x_i)$. We observed these equations as a special case of those for binomial GLMs (but there $y_i$is the proportion of success). The equations are nonlinear and require iterative solution (Adejumo, 2002)

Let X denotes the $N \times p$ matrix of values of $\{x_{ij}\}$. The likelihood equations (24) have form

$$X' = X'\hat{\mu} \tag{25}$$

where$\hat{\mu}_i = n_i \hat{\pi}_i$. This equation illustrates a fundamental result: for GLMs with canonical link, the likelihood equations equate the sufficient statistics to the estimates of their expected values.

The ML estimators $\hat{\beta}$ have a large sample normal distribution with covariances matrix equal to the inverse of the information matrix. The observed information matrix has elements

$$-\frac{\delta^2 L(\beta)}{\delta\beta_a \delta\beta_b} = \sum_i \frac{x_{ia} x_{ib} n_i \exp(\sum_j \beta_j x_{ij})}{[1 + \exp(\sum_j \beta_j x_{ij})]^2} = \sum_i x_{ia} x_{ib} n_i \pi_i (1 - \pi_i) \tag{26}$$

This is not a function of$\{y_i\}$, so that observed and expected information are identical. This happens for all GLMs that use canonical links. The estimated covariance matrix is the inverse of the matrix having elements (26), substituting $\hat{\beta}$. This has form

$$\widehat{cov}\left(\hat{\beta}\right) = \{X' diag[n_i \hat{\pi}_i (1 - \hat{\pi}_i)] X\}^{-1} \tag{27}$$

where $diag[n_i\hat{\pi}_i(1-\hat{\pi}_i)]$ denotes the $N \times N$ diagonal matrix having $[n_i\hat{\pi}_i(1-\hat{\pi}_i)]$ on the main diagonal. This is the special case of the GLM co-variance matrix with estimated diagonal weight matrix $\widehat{W}$ having elements$\widehat{w}_i[n_i\hat{\pi}_i(1-\hat{\pi}_i)]$. The square roots of the main diagonal elements of Eq. (27) are estimated standard errors of $\hat{\beta}$. (Adejumo et al.,2007)

Newton-Raphson method applied to logistic regression

Let
$$u_j^{(t)} = \frac{\partial L(\beta)}{\partial \beta_j}\Big|_{\beta^{(t)}} = \sum_i (y_i - n_i\pi_i^{(t)})x_{ij}$$
$$h_{ab}^{(t)} = \frac{\partial^2 L(\beta)}{\partial \beta_a \partial \beta_b}\Big|_{\beta^{(t)}} = -\sum x_{ia}x_{ib}n_i\pi_i^{(t)}(1-\pi_i^{(t)})$$

Here, $\pi^{(t)}$ approximation t for $\hat{\pi}$, is obtained from $\beta^{(t)}$ through

$$\pi_i^{(t)} = \frac{\exp(\sum_{j=1}^{p}\beta_j^{(t)}x_{ij})}{1+\exp(\sum_{j=1}^{p}\beta_j^{(t)}x_{ij})} \tag{28}$$

We use $u^{(t)}$ and $H^{(t)}$ with formula $\beta^{(t+1)} = \beta^{(t)} - (H^{(t)})^{-1}u^{(t)}$ to obtain the next value $\beta^{(t+1)}$, which in this context is

$$\beta^{(t+1)} = \beta^{(t)} + \left\{X'diag\left[n_i\pi_i^{(t)}(1-\pi_i^{(t)})\right]X\right\}^{-1} X'(y-\mu^{(t)}) \tag{29}$$

where $\mu_i^{(t)} = n_i\pi_i^{(t)}$. This is used to obtain $\pi^{(t+1)}$, and so forth (Haberman, 1988; Adejumo, 2002; 2005).

Results and discussion

Table 1. Proposed and actual number of children

Proposed number of children (A)	**Actual number of children (B)** No child	1 child	2 children	3 children	4 children	≥ 5 children	Total
No child	17	1	7	2	4	4	35
1 child	0	1	2	0	1	1	5
2 children	0	3	24	9	1	1	38
3 children	3	0	7	46	8	2	66
4 children	3	1	1	18	59	6	88
≥ 5 children	3	0	3	9	14	42	71
Total	26	6	44	84	87	56	303

It was observed from Table 1 that, 16.17% of the respondents had above the number of children proposed before marriage when they eventually got married, 21.45% of the respondents had below the number of children proposed before marriage when they got married while 62.37% of the respondents had the exact number of children they proposed before marriage when they got married.

R codes were written for the special loglinear models which are the Quasi-symmetry and Homogeneous Agreement models. Loglinear models have been used to model agreement in terms of components, such as chance agreement and beyond chance agreement by displaying patterns of agreement among raters.[1)]

Quasi symmetry model analysis

Table 2. Parameter estimates under quasi-symmetry model

Coefficients	Estimate Value	Std. Error	z value	Pr(>|z|)
Intercept	3.75395	0.51041	7.355	1.91E-13***
λ_2	1.02942	0.80193	1.284	0.199251
λ_3	1.17514	0.52496	2.239	0.025187*
λ_4	1.26821	0.48848	2.596	0.009425**
λ_5	0.61339	0.47304	1.297	0.194734

λ_6	-0.01628	0.48653	-0.033	0.973314
λ_{56}	-1.79868	0.36277	-4.958	7.12E-07***
λ_{46}	-2.86861	0.46718	-6.140	8.24E-10***
λ_{36}	-3.80806	0.64633	-5.892	3.82E-09***
λ_{26}	-5.08454	1.16408	-4.368	1.25E-05***
λ_{16}	-2.49308	0.47625	-5.235	1.65E-07***
λ_{55}	-0.28980	0.42016	-0.690	0.490368
λ_{45}	-2.18246	0.44042	-4.955	7.22E-07***
λ_{35}	-4.68714	0.82448	-5.682	1.31E-08***
λ_{25}	-4.59683	0.89814	-5.118	3.09E-07***
λ_{15}	-2.85419	0.53564	-5.329	9.90E-08***
λ_{14}	-1.19351	0.46297	-2.578	0.009938**
λ_{34}	-2.89726	0.49990	-5.796	6.80E-09***
λ_{24}	-23.20836	4034.64021	-0.006	0.995410
λ_{14}	-3.66062	0.60549	-6.046	1.49E-09***
λ_{33}	-1.75103	0.55691	-3.144	0.001666**
λ_{23}	-3.94259	0.71428	-5.520	3.40E-08***
λ_{13}	-3.25226	0.59982	-5.422	5.89E-08***
λ_{22}	-4.78337	1.27626	-3.748	0.00178***
λ_{12}	-5.08880	1.18625	-4.290	1.79E-05***
λ_{11}	-0.92073	0.56511	-1.629	0.103247

Goodness of fit statistics (df=10, iteration=16)

G^2 11.914 χ^2 11.07335 AIC= 168.66

Signif.codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

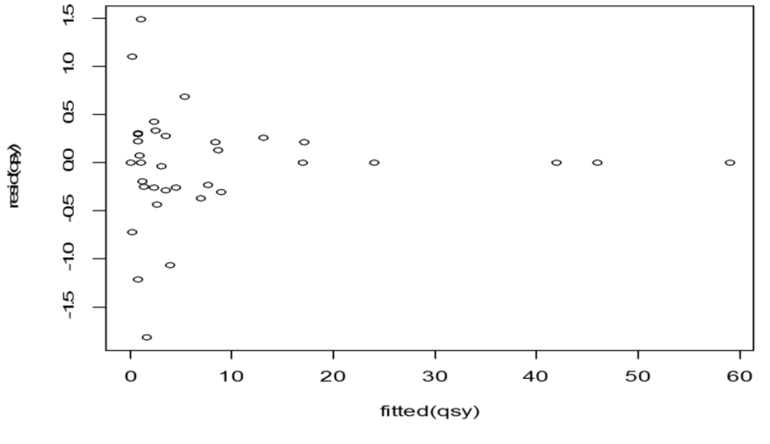

Fig. 1. Fitted quasi symmetry model against residual

` *Homogeneous agreement model analysis*

Table 3. Parameter estimates under homogeneous agreement model

Coefficients	Estimate Value	Std. Error	z value	Pr(>|z|)
Intercept	0.74403	0.22243	3.345	0.000823***
λa_2	-0.95763	0.46946	-2.040	0.041364*
λa_3	0.61232	0.27662	2.214	0.026857*
λa_4	1.14273	0.25456	4.489	7.16E-06***
λa_5	0.95360	0.25439	3.749	0.000178***
λa_6	0.49209	0.26884	1.830	0.067186.
λ_{B2}	-1.69272	0.49033	-3.452	0.000556***
λ_{B3}	-0.18521	0.26371	-0.702	0.482475
λ_{B4}	0.06045	0.24361	0.248	0.804026
λ_{B5}	0.46520	0.23079	2.016	0.043829*
λ_{B6}	0.49969	0.23404	2.135	0.032751*
δ	1.95122	0.12586	15.503	<2E-16***

Goodness of fit statistics (df=24, iteration=5)

G^2 52.324 χ^2 48.07545 AIC = 181.07

Signif.codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

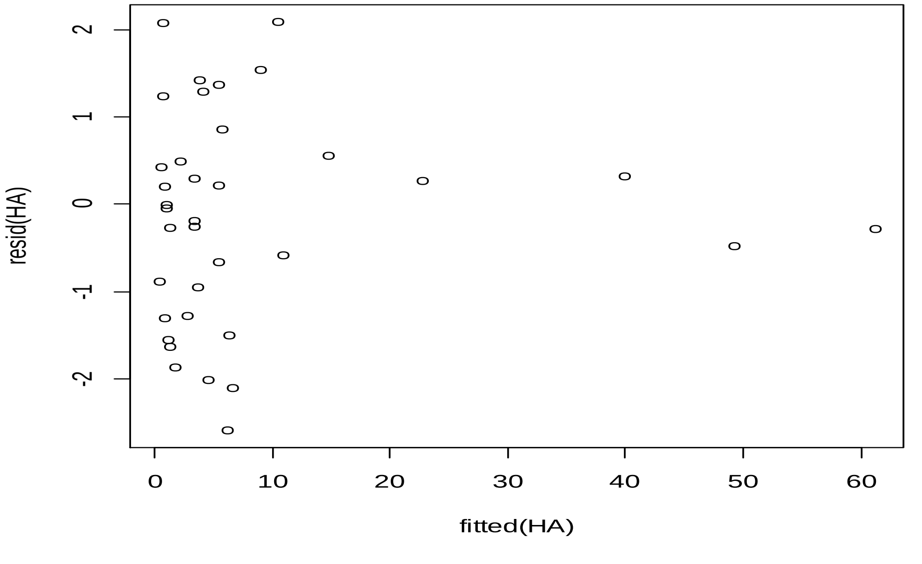

Fig. 2. Fitted homogeneous agreement model against residual

From the results in Tables 2 and 3, and Figures 1 and 2, based on the estimates of goodness of fit as well as graphical illustration of the residual, quasi symmetry model has the better fits for agreement measure than Homogeneous agreement model. Based on the two Figures, (residual versus expected cell counts), we observed that the fitted points are clustered around (close to) zero, it means the model is good. Also using the Akaike's Information Criterion (AIC) quasi symmetry model (QS) has its AIC to be 168.66, while Homogeneous Agreement model has its AIC to be 181.07, so the model with least AIC is better model that fits for agreement of the Man’s proposed and actual number of children.

Table 4. Kappa symmetric measures

	Value	Std. Error	Approx. T	Sig.
Kappa	.520	.035	17.829	<.0001

The estimate for kappa statistic shows the strength of agreement. From Table 4, the kappa value which is 0.520 indicates that there is a strong agreement in the proposed and actual number of children by the man.

The third objective of this study will measure the effect of the independent variables (Age, Religion, Family status, Occupation, Level of education and Ethnic group) on the dependent variable (Actual number of children by the Man).

Table 5. Multinomial logistic regression; model fitting information

Model	Model fitting criteria	Likelihood Ratio Tests		
	-2 Log Likelihood	Chi-square	DF	Sig.
Intercept	550.864			
Final	521.914	28.949	12	.004

In the analysis from Table 5, the probability of the model chi-square (28.949) was 0.004, less than the level of significance of 0.05. This indicates the existence of a relationship between the independent variables (Age, Religion, Family status, Occupation, Level of education and Ethnic group) and the dependent variable (Actual number of children by the Man), the null hypothesis that there was no difference between the model without independent variables and the model with independent variables was rejected.

Table 6. Multinomial logistic regression; likelihood ratio tests

Effect	Model Fitting Criteria	Likelihood Ratio Tests		
	-2 Log Likelihood of reduced model	Chi-square	DF	Sig.
Intercept	527.832	5.918	2	.052
Age	536.694	14.779	2	.001
Family back-	523.393	1.479	2	.477
ground	523.901	1.987	2	.370
Educational status	529.493	7.579	2	.023
Religion	523.871	1.956	2	.376
Ethnic group	523.230	1.315	2	.518
Occupation				

Table 6 shows the relative effects of the independent variables (explanatory variables) to the dependent variable (response variable). It could be deduced that while Age and Religion were significant, Family background, Educational status, Ethnic group and Occupation were not significant.

Table 7. Parameter estimates of the multinomial logistic regression

Actual number of children		B	Std. Error	Wald	DF	Sig.	Exp(B)
Less than 3 children	Intercept	.148	.971	.023	1	.879	
	Age	.062	.164	.145	1	.703	1.064
	Family background	-.240	.337	.508	1	.476	.786
	Education status	.071	.138	.267	1	.605	1.074
	Religion	.173	.310	.311	1	.577	1.189
	Ethnic group	-.299	.277	1.165	1	.280	.742

	Occupation	-.067	.084	.639	1	.424	.935
Greater than 3 children	Intercept	-1.654	.853	3.756	1	.053	
	Age	.472	.143	10.860	1	.001	1.603
	Family background	.131	.290	.206	1	.650	1.141
	Education status	-.096	.119	.656	1	.418	.908
	Religion	.676	.268	6.356	1	.012	1.966
	Ethnic group	.031	.213	.021	1	.884	1.032
	Occupation	.020	.072	.077	1	.781	1.020

The reference category is: mumber of child equals three

From Table 7, the independent variables age and religion are significant in distinguishing between men whose actual number of children is greater than 3 and those whose actual number of children equals three.

For each unit increase in age, the odds of a man having more than 3 children increases by 60.3% (1.603-1), for each unit increase in religion, the odds of a man having more than 3 children increases by 96.6% (1.966-1).

The fourth objective of this study measures a probability model for a newly wedded couple ever having a baby boy. From the data, the probability (p) of having at least one male child is 0.8019. The chance of a newly wedded couple ever having a male child at any trial follows a Geometric Probability Distribution.

$$f(x) = (0.8019)(0.1981)^{x-1}, x = 1,2,3\ldots$$

Table 8. Various probabilities of having a male child

x	$f(x) = (0.8019)(0.1981)^{x-1}$
1	0.8019
2	0.1589
3	0.03146
4	0.006234
5	0.001234
6	0.0002446
7	0.00004846
8	0.00000960
9	0.00000190
10	0.000000376

From Table 8, it could be deduced that as the number of trial increases, the probability of having at least a male child decreases reason being that reproduction power by the man reduces on the average as age increases and hence makes the couples sexual intimacy to lack luster.

Conclusion

Based on the research so far, 16.17% of the respondents had above the number of children proposed, 21.45% of the respondents had below and 62.37% had exact number of children they proposed before marriage. We observe that Quasi symmetry is better than the Homogeneous Agreement model in describing the pattern of Association that exists between the proposed and actual number of children by the man.

We also observed that the estimate of the probability of having at least one boy is 0.8019 based on the data collected and that age and religion of the man influenced the actual number of children in such family.

NOTES

1. http://www.gbif.org/resource/81287

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