MATHEMATICS TEACHERS' RESPONSES AND PERCEPTIONS ON PAPER FOLDING ACTIVITIES IN TEACHING MATHEMATICS

Mulugeta ATNAFU
Addis Ababa University, ETHIOPIA

Abstract. The purpose of this study was to examine Addis Ababa city schools' mathematics teachers' responses on the paper folding activities and perception on implementing paper folding in teaching mathematics. Exploratory survey design and quantitative and qualitative research methods were used. 75 mathematics teachers were selected by cluster and simple random sampling. Training manual activities and questionnaire were used for data collection, and analysed using mean, standard deviation, independent samples t-test and qualitative analysis. Some of the findings were the primary school teachers have better perceptions than the secondary school teachers and male teachers have better perceptions than female teachers in the importance of applying paper folding in teaching mathematics than female teachers. The primary and secondary school teachers and male and female teachers have almost similar challenges in implementing paper folding in teaching mathematics. In addition, some of the teachers challenged in drawing of perfect figures, cutting or folding the papers to get the properties of mathematical expressions. Therefore, some of the recommendations were training on paper folding should be given at the beginning of the year for mathematics teachers; paper folding is included in the design of the curriculum; and necessary materials should be provided.

Keywords: paper folding, perception, teaching mathematics, interaction

Introduction

Paper folding is an effective tool in teaching mathematical concepts, particularly in geometry, algebra and calculus (Boakes, 2006; Georgeson, 2011; Wares, 2011). According to Brady (2008), applying paper-folding activities in teaching primary and secondary mathematics can achieve the goals for teaching and learning mathematics by engaging students effectively in mathematical modelling, visualising, algebraic thinking and problem-solving. Paper-folding activities can provide a hands-on activities and active participation of students in solving and understanding mathematical concepts and develop skills in mathematical thinking; communications; and students' interaction. Using both hands working together, the students learn how to manipulate the paper and carry out certain steps repeatedly in order reach the suggested result, and these actions require memory and psychomotor skills in learning mathematics and real-life applications (Coad, 2006).

Paper folding has educational, social and affective values in teaching-learning of mathematics. In the educational values, paper folding contributes to the development of mathematical ideas and thinking, and leads to understanding of the mathematical concepts (Cornelius & Tubis, 2009); spatial visualisation and mathematical reasoning (Georgeson 2011); and enhance the problem-solving skills (Youngs & Lomeli, 2002). The value of paper-folding in teaching mathematics identified by Sze are paper folding promote the correct use of geometric terminology, enhance the students' critical thinking, create students' dialogue and encourages cognitive development (Sze, 2005). Paper folding in teaching mathematics requires precision and neatness for understanding mathematical concepts, which will demand students' focus and use their motor skills.

In the social values, paper folding has the potential to teach students in cooperative learning. Paper folding activities encourage students communicate and interact with others. It helps to teach a group of students how to use their individual skills, sharing feelings and knowledge, breaking down barriers, have positive social interaction, promotes co-operation and well-being.[1)] Paper folding activities also promote student engagement in mathematical learning (Brady, 2008). Paper-folding tends to eliminate the status associated with age differences; the younger students are often in a position to teach the older students, and it provides an activity that works well when teaming different grade levels.

In the affective values, paper folding in teaching mathematics promotes positive emotions that appear in terms of sense of pleasure brought on simply by understanding what is being done. Schlöglmann (2002) refers to the implicit emotional memory system that can be activated by the problem solving process and encourages self-belief. Mathematics learning using paper folding hands-on activities where the atmosphere in the classroom is more relaxed, increase student interest and motivation, and promote self-esteem.[2)] Paper folding provides for the students as an opportunity of fun, develops a hobby, and promotes a feeling of achievement and well-being.[1)]

Statement of the problem

The Ethiopian National Examination Agency indicated that students' achievement in grade 8 had influenced by the major factors as teachers' attitude explained 32.5% and student background variables as independent group explained 36.6% of the variation in learners' achievement. From several variables or factors that contribute for students' achievement considered in the study, 32.5% of the contribution for students' achievement was by teachers' attitude and 36.6% of the contribution was by student background variables. It is not difficult for a teacher to continuously make mathematics interesting for their students. Many students find mathematics boring, and one great way to make mathematics interesting is applying paper folding.[1)]

Motivation for this research arises as a response to one of the central issue recognized nationally, that the current trend is not on the way of achieving its goal the decline in achievement, the need for developing new pedagogical approaches to improve science and mathematics instruction (Eshetu et al., 2009). Some educators convinced about the value of paper folding activities in the classroom to support the development of mathematical understanding.

Research questions

The purpose of this study was to examine Addis Ababa city primary and secondary schools of mathematics teachers' responses on the paper folding activities and perception on implementing paper folding in teaching mathematics. Here are the research questions that the study would answer: (1) how do the teachers understand the paper folding activities in the two days of training; (2) what are the perceptions of the teachers on the importance of implementing paper folding in teaching mathematics; (3) what is the significant difference in the perceptions of the teachers on the importance of implementing paper folding in teaching mathematics with respect to level of teaching and gender; (4) to what extent the teachers implemented or will implement paper folding in teaching mathematics; (5) what are the challenges of implementing paper folding in teaching mathematics; (6) what is the significant difference in the challenges of implementing paper folding in teaching mathematics with respect to level of teaching and gender.

Materials and methods

Research design

Mixed design such as survey design and a single subject quasi-experimental design were used in this study. In the survey design data were collected from a large area such as 15 schools and 75 teachers, and in the single subject quasi-experimental design the selected mathematics teachers assigned in one group and a two days training on paper folding as a treatment was given. The data were collected after two days of training through quantitative data collection using rating scales and qualitative data collection through open ended questionnaire and paper folding activities.

Population and sampling techniques

The population of the study were the primary and secondary schools mathematics teachers of Addis Ababa city. Eight secondary schools and seven primary schools were selected using cluster sampling and five mathematics teachers taken by using simple random sampling from the selected schools a total of 75 teachers participated in the study of paper folding in teaching mathematics. Out of 75 teachers 35 were from primary school and 40 from secondary school; and 44 were male and 31 female teachers.

Instruments of data collection

In this study, training manual activities and questionnaire were used for data collection. The activities in the training manual were designed to give clarifications and open ended questions. The activities divided in seven sections with a total of 34 activities. The activities were divided in different areas such as: properties of quadrilateral; properties of triangles; properties of circles; finding the areas; showing some algebraic expressions; solving some mathematical equations; and constructing solids figures. The questionnaire asked the participant teachers as to what extent the teachers perceive the importance of implementing paper folding, what are the challenges of implementing paper folding, and to what extent the teachers implemented or will implement paper folding in teaching mathematics.

Validity and reliability of the instruments

The questionnaire was reviewed based on the comments of professionals in order to achieve the face and content validity. A pilot study was conducted to determine the validity and reliability of the perception scales. Thirty in-service trainers of Addis Ababa University were chosen by simple random sampling for the pilot study. The alpha coefficient of Cronbach yielded 0.836 for the perceptions on the importance of implementing paper folding scale, and 0.783 for the challenges of implementing paper folding scale. Cronbach Alpha coefficients of reliability for the two perception scales indicated that they have high internal-consistency reliability.

Procedure of the data collection

Two days of training were arranged for mathematics teachers who teach in the primary and secondary schools of Addis Ababa city. The 75 teachers who teach mathematics in primary and secondary schools divided into 10 groups of which each group contains between 7 and 8 teachers. A handout containing the procedure of paper folding activities in middle secondary (grade 7 to 10) geometry and algebra contents distributed to each of teachers. The teachers work in groups and reflect their findings to the whole class.

Data analysis techniques

The skewness values of all the Likert scale items on the importance of paper folding in teaching mathematics and the challenges of applying paper folding in teaching mathematics were between -1 and 1. Accordingly, the data distribution is approximately normal. To this effect, descriptive and inferential statistics such as mean, standard deviation and independent samples t-test were employed. In addition, qualitative analysis employed for the data obtained from the open ended questionnaires asked the respondents about their challenges in applying paper folding in teaching mathematics and the responses of the reflections of the activities given in the class of the training.

Results

Result from observation of trainee teachers' reflection

The first research question was: how do the teachers understand the paper folding activities in the two days of training? In the training manual activities clarifications and open ended questions were designed. The activities were divided in to seven sections with a total of 34 activities. Below are the some of the responses of the group of teachers for the selected activities given in the training:

Some of the activities given to the teachers from each section discussed below. In section one of the properties of quadrilateral, one of the activities was: half of the sum of the two opposite parallel lengths of a parallelogram is the same as the length of the line segment joining the mid-points the non-parallel lengths. The procedures on the activity is explained to the teachers as: draw any trapezoid ABCD and cutting out; fold the altitudes at both ends of the shorter base of the trapezoid ABCD; bisect each nonparallel side and connect these midpoints with a crease EF; and fold lines perpendicular to AB through E and F.

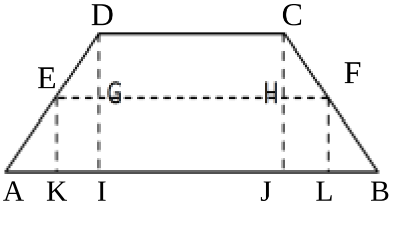

Figure 1. Properties of a quadrilateral

The researcher gave enough time for each groups to draw, cut and fold, and all groups of teachers did the four procedures given above within a given time. Then the researcher asked the groups of teachers the first question to compare DG and CH with GI and HJ respectively by folding along EF. The different groups answered differently as: DG=CH and GI=HJ; or DG=GI and CH=HJ; or DG=½DI and CH=½CJ; or DC=½EF and EF=½AB, etc depending on the teachers visualized it. All the answers were correct, but in terms of the question they are asked the exact answer is DG=GI and CH=HJ. The second question asked was: what are the images of DG and CH in a reflection in EF? They gave different answers such as: GI and HJ respectively; or I and J respectively; or IJ, etc. The groups who gave the first answer were correct. The others may make mistakes due to the misconception of ‘reflection’, they are not clearly understood what reflection mean. The third question asked was: how does the sum of CD and AB compare with the median EF? They gave all the answers CD+AB=2EF correctly, but they differ the reason they gave. Some gave the answer by guessing without giving any reason; some gave the reason as E and F are midpoints AD and BC respectively; and others gave the reason by folding and showing as CD=GH=IJ, AK=KI=EG and LB=JL=HF, and CD+AB=CD+(AK+KI+IJ+JL+LB)=CD+IJ+(AK+KI+JL+LB)=2GH+2EG+2HF=2EF. It seems the last one was the best reasoning given by the groups for the answer.

In section two of the properties of triangles, one of the activities was: the angle bisectors of a triangle intersect at a common point. The procedures given by the researcher were: draw any triangle ABC and cutting out; fold the bisectors of each angle of the given triangle; and fold the perpendiculars from this point of intersection to each of the sides of the triangle.

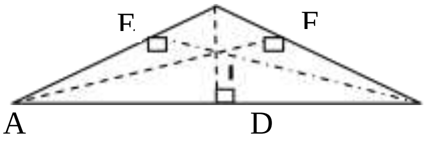

Figure 2. Showing the angle bisectors of a triangle intersect at a common point

All the groups of teachers well did the procedures above within specified time given. Then the groups of teachers asked to answer the first question as: Do the bisectors intersect in a common point? What is the point of intersection of the angle bisectors called? Some groups of teachers didn't show intersect at one point, but they know this property in their teaching in the classrooms that the bisectors of the angles meet at one point. The point they answered was in centre. But the difficulties in most groups were perfection in measuring and folding. If measuring and folding of the procedures were not perfectly done, then they could not get the intersection is at one point. The second question raised was to compare the sides ID, IE, and IF by folding. Some groups did well the first question and they got the answer that three sides ID, IE, and IF are equal. Other groups did not well do the first question and they got difficulty in showing they are equal. The groups did not well do the activities understood how the properties work using paper folding by observing the presentation of the well did groups.

In section three of the properties of circles, one of the activities was: the arcs of a circle intercepted by parallel lines are equal. The procedures given by the researcher were: draw any circle and cutting out; fold any diameter AB of circle O; and fold two chords, each perpendicular to AB.

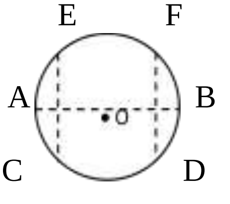

Figure 3. Showing the arcs of a circle intercepted by parallel lines are equal

The groups of teachers drown a circle using either protractor or cents or folding a paper and pointing a hole on the two edges where the first hole is the fixed point (center) pressed by pin and the second hole is rolled by using pencil. Next they asked to answer the question to compare arc EF to arc CD by folding.

There was no problem observed in drawing and cutting out, but some difficulties observed in some groups of drawing two perpendicular lines to AB. Some showed the exact relationship as arcs EF and CD are equal. Some did not show it because of lack of perfection of drawing two lines perpendicular to AB.

In section four of finding the areas of plane figures, one of the activities given to the teachers was: derive the area of a circle. The procedures explained to the teachers were: draw any circle with radius r and cutting out; fold it into two haves and colour one half; fold again to form quarters and again and again to form eighths and then sixteenths; and cut out each sectors and arrange up and down to form a parallelogram.

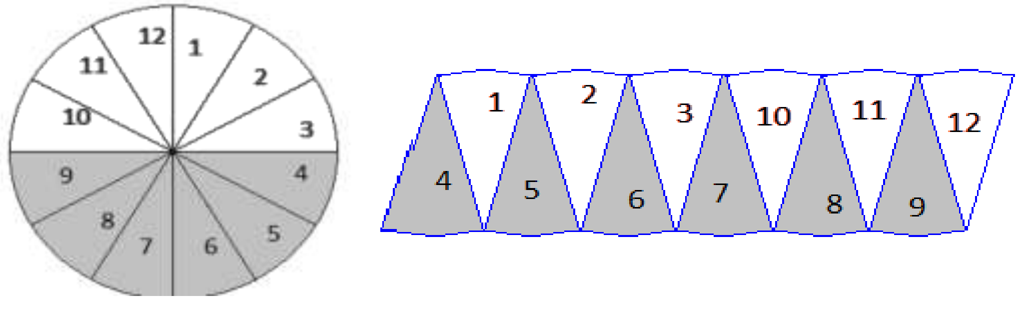

Figure 4. Deriving the area of a circle to be πr

The groups of teachers drown a circle using either protractor or cents or folding a paper and pointing a hole on the two edges, and drawing a diameter. Then the groups of teachers asked a question as: What is the length of a base of the parallelogram? Some of the teachers tried to measure with a ruler to find the length of the base. The correct answer given by some groups as ½ of the circumference of the circle (πr), since the sum of the two lengths of bases of the parallelogram is the same as the circumference of a circle. The second question asked was: What is the height of the parallelogram and its approximate area? Some groups of teachers answered quickly the height is r and its area by calculating as: $A=\pi r\times r=\pi r^2$. They gave the reasons as the height of the parallelogram is the radius of the circle and the area of a circle is equivalent to the area of the parallelogram as A=base×height=$\pi r \times r = \pi r^2$. Others groups of teachers understood by the reason given by the group of teachers.

In section five of showing some algebraic expressions, one of the activities was: illustrating the identity $(a+b)(a-b)=a^2-b^2$. The procedures given by the researcher were: draw any square of length a and cutting out; draw a square of length b (b<a) at one edge of the square (X); fold the three rectangles X, Y and Z as shown in the figure and cutting out; and remove X and rearrange a rectangle Y and Z as in the second diagram.

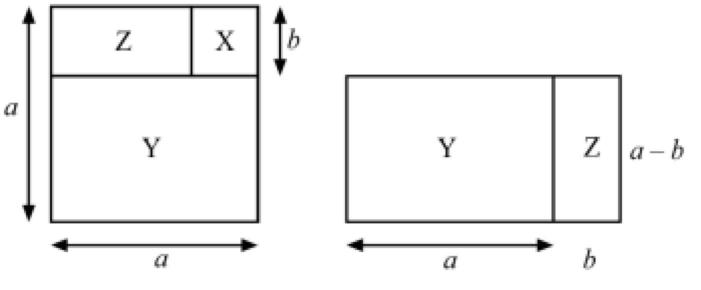

Figure 5. Illustrating the identity $(a+b)(a-b)=a^2-b^2$

The groups of teachers took different lengths of squares in their drawings, cutting out, and arranging the cut out to form a new rectangle with the given specified time. The groups of teachers asked a question as: What is the area of the regions Y+Z? Some of the teachers answered as (a+b)(a–b) and others said as 'it is equal to the area enclosed by the larger square (with side of length a) minus the area enclosed by the smaller square (with side b)'. The two answers given by different groups are correct. Hence the area of the region Y+Z is numerically equal to a^2-b^2. The diagram on the right is a restructuring of the first diagram. The area of the region Y+Z is now that of a rectangular region with sides of length (a+b) and (a–b) respectively. Hence the area measure is numerically equal to (a+b)(a–b). This illustrates that $(a+b)(a-b)=a^2-b^2$. All the groups of teachers understood clearly the algebraic expressions using paper folding.

For section six of solving some quadratic equation, $x^2–px+q=0$, where p and q integers using paper folding, the following procedures communicated: *f*old two intersecting lines, X'X and Y'Y, intersecting at O; coordinatize each of the lines by folding equally spaced points; let OP and OQ represent p and q respectively, and fold perpendiculars to X'X and Y'Y at P and Q, intersecting at M; fold a line determined by M and U, where OU is the line representing +1; find the midpoint of UM by folding (let T be this midpoint); reflect U in some lines that passes through T so that the image of U is on X'X; and If these two points are R and S on X'X of the two lines intersect on the x-axis, then OR and OS represent the roots in both magnitude and sign.

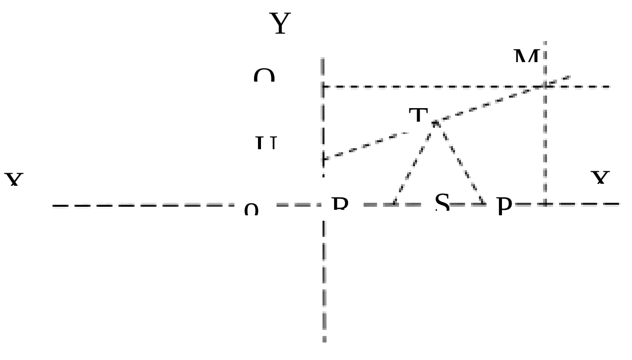

Figure 6. Solving quadratic equation of the form $x^2–px+q=0$

All the groups of teachers well did the procedures above within specified time given. Then the groups of teachers asked to find the two real roots of $x^2–5x+6=0$. Only one group answered the answer of the two real roots by folding paper as: OR=2 and OS=3. Some of the groups of teachers challenged in drawing perpendicular lines, some of the groups of teachers challenged in coordinatizing the perpendicular lines with equally space points, most of the groups of teachers challenged in getting the exact intersection point M, but for all nine groups challenged to get the exact solution sets by reflecting U in some lines that passes through T so that the image of U is on X'X. But it took more time for all the groups of teachers to understand the procedures of paper folding activities to get the solutions.

Result from the questionnaire on the perception of trainee teachers

The second research question was: what are the perceptions of the teachers on the importance of implementing paper folding in teaching mathematics? Below is the descriptive statistics of the perception of the teachers on the importance of paper folding in teaching mathematics.

Table 1. Descriptive statistics of the perception of teachers on the importance of paper folding in teaching mathematics

Items on the importance of teaching mathematics using paper folding	N	M	SD
It helps better understanding of students in mathematical concepts	75	4.79	.42
It helps in problem solving of students in mathematics	75	4.54	.72
It helps for critical thinking of students in mathematics	75	4.71	.46
It enhances the reasoning power of the students in mathematics	75	4.42	.58
It improves the creativity of the students in mathematics	75	4.79	.42
It improves the achievement of students in mathematics	75	4.63	.58
It increase social interaction between students	75	4.50	.78
It enhance the close contact between teacher and students	75	4.42	.93
Mathematics is enjoyable using paper folding	75	4.75	.44
It decreases the anxiety of students in learning mathematics	75	4.21	.88
It develop the confidence of the students in learning mathematics	75	4.67	.48
It improves the attitude of students in learning mathematics	75	4.46	.51
It enhances the motivation of students in learning mathematics	75	4.79	.51

Regarding to the cognitive development, Table 1 indicate that most of the teachers agreed that paper folding help better understanding (mean=4.79), enhancing critical thinking (mean=4.71); improving the creativity (mean=4.79); improving the problem solving (mean=4.54), enhancing the reasoning power (mean=4.42), and improving the achievement (mean=4.63) of the students in mathematics. Regarding to social factor, most of the teachers also agreed that paper folding increasing the social interaction between students (mean = 4.50) and enhancing the close contact between teacher and students (mean=4.42). Regarding to affective factor, most of the teachers agreed that paper folding helps mathematics enjoyable (mean=4.75), developing the confidence (mean=4.67) and improving the attitude (mean=4.46); enhancing the motivation (mean=4.79); and reducing the anxiety (mean=4.21) of students in learning mathematics.

The third research question was: What is the significant difference in the perceptions of the teachers on the importance of implementing paper folding in teaching mathematics with respect to level of teaching and gender? Whether this difference is statistically significant or not, an independent sample t-test conducted and the result of which is presented in the following table.

Table 2. Comparison between the mean score of teachers' responses on importance of teaching mathematics using paper folding by level of teaching and gender

Item	Variables	Group	*N*	*M*	*SD*	*t*	*df*	*P*
Importance of teaching mathematics using paper folding	Level of teaching	Primary	35	4.88	.950	2.666	73	.01
		Secondary	40	4.32	.870			
	Gender	Male	44	4.72	.887	1.568	73	.02
		Female	31	4.38	.981			

From the teachers' response of an independent t-test, the two mean scores of primary and secondary school teachers have found to be statistically significant (*t*=2.666, *df*=73, *P*=.01). From these the primary school teachers found to outperform the secondary school teachers on perceptions of the importance of implementing paper folding in teaching mathematics at a better level. Similarly, the difference between gender found to be statistically significant (t=1.568, df=73, P=.02) and conclude that male teachers found to have better perceptions of the importance of implementing paper folding in teaching mathematics than female teachers.

The fourth research question was asking to what extent the teachers implemented or will implement paper folding in teaching mathematics. Below is the descriptive statistics of the perceptions of teachers for the extent of implemented paper folding previously and want to implement it in the future in teaching mathematics.

Table 3. Descriptive statistics for the extent of implemented paper folding previously and want to implement it in the future in teaching mathematics

Variables	N	M	SD
To what extent you were implemented paper folding in teaching mathematics	75	2.13	.797
To what extent do you want to implemented paper folding in teaching mathematics in the future	75	3.75	.737

Table 3 indicate that most of the teachers reported that they were not implementing paper folding in their teaching mathematics (mean=2.13) and also reported as they will implement paper folding in teaching mathematics in the future (mean=3.75).

The fifth research question was asking to identify the challenges of implementing paper folding in teaching mathematics. Below is the descriptive statistics of the perception of teachers on the challenges of implementing paper folding in teaching mathematics.

From the Table 4, the teachers agreed about the challenge of implementing paper folding in teaching mathematics were: shortage of time (mean=3.63), heavy workload (mean=3.38), lack of sufficient knowledge (mean=2.63), lack of sufficient skill (mean=2.42), shortage of material (mean=2.29), didn't have interest (mean=2.17), students didn't have interest (mean=2.54), and paper folding will not help in improving teaching-learning of mathematics (mean=1.50). The most challenge in implementing paper folding in teaching mathematics were shortage of time and heavy workload; next at the middle of challenges were teachers didn't have sufficient knowledge and skills in applying paper folding, and students didn't have interest; and the least challenges were shortage of materials, teachers didn't have interest, and paper folding will not help in improving teaching-learning of mathematics.

Table 4. Descriptive statistics of the perception of teachers on the challenges of implementing paper folding in teaching mathematics

Items on the challenges of applying paper folding in teaching mathematics	**N**	**M**	**SD**
I have heavy workload	75	3.38	1.44
I didn't have sufficient knowledge	75	2.63	1.56
I didn't have sufficient skill	75	2.42	1.50
Shortage of time	75	3.63	1.17
Shortage of materials	75	2.29	1.33
I didn't have interest	75	2.17	1.37
Students didn't have interest	75	2.54	1.38
It will not help in improving teaching-learning of mathematics	75	1.50	1.02

The sixth research question was: What is the significant difference in the challenges of implementing paper folding in teaching mathematics with respect to level of teaching and gender? Whether this difference is statistically significant or not, an independent sample t-test conducted and the result of which is presented in the Table 5.

From the teachers' response of an independent t-test, the result found to be statistically non-significant for level of teaching (t=-.763, df=73, P=.62) and gender (t=.487, df=73, P=.84). From this the primary and secondary school teachers have almost similar challenges in implementing paper folding in teaching mathematics. Similarly, male and female teachers also have almost similar challenges in implementing paper folding in teaching mathematics.

Table 5. Comparison between the mean score of teachers' responses on challenges of implementing paper folding in teaching mathematics by level of teaching and gender

Item	Variables	Group	*N*	*M*	*SD*	*t*	*df*	*P*
Challenges of applying paper folding in teaching mathematics	Level of teaching	Primary	35	2.51	.724	-.763	73	.62
		Secondary	40	2.63	.638			
	Gender	Male	44	2.61	.813	.487	73	.84
		Female	31	2.52	.756			

Discussion

From the finding, most of the teachers agreed that paper folding help to enhance the understanding, critical thinking, creativity, problem solving, reasoning power, achievement, enjoyment, confidence, attitude, motivation and reduces the anxiety of the students in learning mathematics, improves social interaction between students and close contact between teacher and students. The primary school teachers have better perceptions than the secondary school teachers and male teachers have better perceptions than female teachers on the importance of implementing paper folding in teaching mathematics. Regarding to cognitive development, the findings are in line with the findings of Coad (2006) indicated that paper folding encourages cognitive development; Cornelius & Tubis (2009) showed that paper folding contribute to the development of mathematical ideas and thinking, and the understanding of mathematical concepts; Youngs &Lomeli (2002) found that paper folding enhancing problem-solving skills; and Fenyvesi et al. (2014) indicated that paper folding helps in solving a problem in different perspectives, improve problem solving and reasoning skills of the students. Regarding social value, the findings are in line with the finding[1] showed that paper folding help students in teaching cooperative learning, learning how to communicate and interact with others, encourages students to help others, eliminate the status associated with age differences, provides an activity that works well when teaming different grade levels, helps the students in team building exercise, sharing feelings and knowledge, breaking down barriers, have positive social interaction, and promotes co-operation and well-being; and Sze (2005) showed that paper folding encourages create a class dialogue; and Tubis & Mills (2006) discussed paper folding promotes the development of fine motor skills and manual dexterity. In the affective aspects, the findings are in line with the findings of Schlöglmann (2002) showed that paper folding helps to have positive emotions; Budinski[2] indicated that paper folding increase student interest and motivation; and stimulates children, an opportunity for fun and can be developed as a hobby, and[1] showed that paper folding promotes a feeling of achievement and well-being. Boakes (2006) indicated that the factors contributing to these result males often more confident than their female counterparts.

Most of the teachers reported as never or rarely applied paper folding in teaching mathematics, but they will apply paper folding in teaching mathematics in the future. This result indicates that even some of the teachers have doubt in applying it in the future because of the following challenges they have. The first challenge of the teachers replied were: shortage of time for preparation, high work load and vast content of the curriculum. The shortage time for preparation may be the teachers load is 30 periods per week (i.e., 6 periods per day) and working with different committees of the school activities. Therefore, the teachers are busy inside and outside the classrooms and the shortage of time for covering the content is due to vast content of mathematics made challenging for covering the contents. The second reason replied was lack of motivation of the teachers in the teaching profession and interest of the students to learn mathematics. The third reason was teachers did not have deep knowledge and skill about paper folding and they need awareness and training. The fourth reason they mentioned was large class size and shortage of materials. The class size range from 50 to 80 students per class and the school didn't provide paper, scissors and rulers for the paper folding activities.

But paper folding has many applications in teaching primary and secondary mathematics, especially in geometry and algebra. It is important in teaching symmetry; for many of the folds, whatever is done to one side is done to the other. This is, of course, a fundamental algebraic rule that can be shown outside the framework of a formal "math lesson". In addition, it is also showed paper-folding allows students to create and manipulate basic geometric shapes.[1)] Paper folding can also be used to solve problems that have been interesting in the context of Euclidean constructions with straightedge and compass. It is easy to trisect an angle using paper folding and also to find the cube root of two (among other numbers) – that is, to double the cube. Coad (2006) indicated that paper folding also enables students to visualise parabolas and other conics as folded structures. Its use can be extended to problems that are interesting in their own right, including folding rational angles and star polygons.

The most apparent evidence of affective engagement associated with the paper-folding activities was the sense of enjoyment that the teachers expressed. All the teachers motivated and exited in the paper folding application in the teaching mathematics. But some challenges observed were: misconception of some concepts; guessing without reasoning; lack of perfection in measuring and folding; drawing a line perpendicular to another line; lack of perfection in drawing; shortage of time, heavy workload, lack of sufficient knowledge, lack of sufficient skills, shortage of material, lack of students and teachers interest. The suggestions given by the teachers to apply paper folding in teaching mathematics were: the load of the teacher should be adjusted from 30 period per week to at least 20 periods per week; the teachers' salary must be proportional to his work in order to improve the motivation of the teachers; training on paper folding should be given at the beginning of the year for mathematics teachers in order to improve the knowledge and skills of the teachers to apply paper folding in teaching mathematics; since the textbook of mathematics is not included paper folding as a teaching tools, it should be included in the design of the curriculum; and schools provide necessary materials like paper, ruler and scissors.

Conclusions and recommendations

In conclusion paper folding has many applications in teaching primary and secondary mathematics, especially in geometry and algebra. Teaching mathematics using paper folding improves the understanding, problem solving, critical thinking, reasoning power, creativity, achievement of students in mathematics, the interaction between students and between teacher and students; but decreases the anxiety of students in learning mathematics and improves the enjoyment, confidence, attitude and motivation of students in learning mathematics. Teaching mathematics using paper folding have many challenges, for the learners side challenges in drawing of perfect figures, cutting or folding the papers to get the properties of mathematical expressions, and from teachers side challenges like shortage of time and heavy workload, they didn't have sufficient knowledge and skills in applying paper folding, and students didn't have interest, shortage of materials they were not applying paper folding frequently in teaching mathematics in the classroom. One of the important aspects of applying paper folding in teaching-learning mathematics is providing open ended activities; students participated in group discussion, questioning, explaining, debating, presenting and feedback.

The recommendations of the study were: (A) encouraging teachers to use paper folding in teaching mathematics; (B) training on paper folding should be given at the beginning of the year for mathematics teachers; (C) paper folding is included in the design of the curriculum; (D) necessary materials like paper, ruler and scissors should be provided.

NOTES

1. http://home.earthlink.net/~robertcubie/origami/edu.html
2. https://sciforum.net/manuscripts/2882/manuscript.pdf

REFERENCES

Boakes, N.J. (2006). *The effects of origami lessons on students' spatial visualization skills and achievement levels in a seventh-grade mathematics classroom: DEd thesis.* Philadelphia: Temple University.

Brady, K. (2008). Using paper-folding in the primary years to promote student engagement in mathematical learning (pp. 77-83). In: Goos, M., Brown, R. & Makar, K. (Eds.). *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Canberra: MERGA.

Coad, L. (2006). Paper folding in the middle class room and beyond. *Australian Math. Teacher*, *62*(1), 6-13.

Cornelius, V. & Tubis, A. (2009). On the effective use of origami in the mathematics classroom (pp. 507-515). In: R. Lang, R.J. (Ed.). *Origami 4: fourth iternational meeting of origami science, math, and education* Natick: A. K. Peters.

Eshetu, A., Dilamo, O., Tesfaye, A. & Zinabu, M. (2009). Science and mathematics secondary education in Ethiopia: an option paper presented at the technical workshop on science and mathematics secondary education in Africa. Tunis.

Fenyvesi, K., Budinski, N. & Lavicza, Z. (2014). Two solution to an unsolvable problem: connecting origami and GeoGebra in a Serbian high school. *Proc. Bridges: Mathematics, Music, Art, Architecture, Culture,* pp. 95–102.

Georgeson, J. (2011). Fold in origami and unfold math. *Math. Teach. Middle School, 16*, 354-361.

Schlöglmann, W. (2002). Affect and mathematics learning (pp. 185-192). In: Cockburn, A.D. & Nardi, E. (Eds). *Proceedings of the 26th conference of the international group for the psychology of mathematics education.* Norwick: University of Norwich.

Sze, S. (2005). *Math and mind mapping: origami construction*. Dunleavy: Niagara University.

Tubis, A. & Mills, C. (2006). *Unfolding mathematics with origami boxes.* Emeryville: Key Curriculum Press.

Wares, A. (2011). Using origami boxes to explore concepts of geometry and calculus. *Int. J. Math. Educ. Sci. & Tech., 42*, 264-272.

Youngs, M. & Lomeli, T. (2002). *Paper square geometry: the mathematics of origami.* Fresno: AIMS Education Foundation.

✉ Dr. Mulugeta Atnafu
Department of Science and Mathematics Education
Addis Ababa University
P.O. Box 3155, Addis Ababa, Ethiopia
E-Mail: ayatmu@yahoo.com