MATLAB SUPPORTED LEARNING AND STUDENTS' CONCEPTUAL UNDERSTANDING OF FUNCTIONS OF TWO VARIABLES: EXPERIENCES FROM WOLKITE UNIVERSITY

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Abstract. A non-equivalent groups quasi-experiment research and case study designs were conducted at Wolkite University to investigate MATLAB supported learning and students' conceptual understanding in learning Applied Mathematics II using four different comparative instructional approaches: MATLAB supported traditional lecture method, MATLAB supported collaborative method, only collaborative method and only traditional lecture method. Four intact classes of Mechanical Engineering groups 1 and 2, Garment Engineering and Textile Engineering students were selected by simple random sampling out of eight departments. The first three departments were considered as treatment groups and the fourth one "Textile Engineering" was assigned as a comparison group. The Departments had 30, 29, 35 and 32 students respectively. The results of the study showed that there is a significant mean difference on students' conceptual understanding between groups of students learning through MATLAB supported collaborative method and the other learning approaches. Students who learnt through MATLAB technology supported learning in combination with collaborative method were found to understand concepts of functions of two variables better than students learning through the other methods of learning. These, hence, are informative of the potential approaches universities would follow for a better students' understanding of concepts.

Keywords: MATLAB supported collaborative method, MATLAB supported learning, collaborative method, conceptual understanding, functions of two variables

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Introduction

Mathematics is one of the widely offered courses to engineering and technology students at university level in different areas of study. Among those courses Applied Mathematics I, II and III are the major ones. Those courses at university level are being taught by professional mathematics teachers who do not all seem to observe that there is a problem of communication between them and their students who study engineering and technology (De Guzmán et al., 1998; Maull & Berry, 2001). It is most of the time observed that engineering and technology students lack a foundational base of mathematics that is conceptual understanding.

According to Maull & Berry (2001) mathematics students are more performing conceptual understanding in mathematics when compared with engineering and technology. However, engineering and technology students apply mathematical concepts in their daily activities and particularly in engineering and technology courses as compared with mathematics students. So, in order to tackle poor conceptual understanding of these students, there are researches that highly recommend the use of instructional method that enables the students to discuss with one another and lets them to construct their own understanding in general and collaborative method in particular (Wong, 2001). There are also some researches recommending utilization of different mathematical software to develop students' understanding (Atnafu et al., 2015).

According to Bekele (2012) majority of higher institutions in Ethiopia employ traditional lecture method. Traditional lecture method is inadequate in developing a deep conceptual understanding. In the traditional method of teaching more focus is given to formula based problem solving rather than acknowledging the concepts behind (Antwi, Hanson, Savelsbergh, & Eijkelhof, 2011). Literature indicates that, learning mathematics using symbolic packages enable students to achieve a high level of reasoning by visually supporting the concepts with graphical representations (Drijvers, 2000; Kendal & Stacey, 2002; Kramarski & Hirsch, 2003; Perjesi, 2003; Peschek& Schneider, 2002). Nowadays, there is widely available software to be used for the purpose of teaching. Particularly, for advanced mathematics courses like Applied Mathematics II, MATLAB is used to visualize and plot different 2D and 3D graphs for better understanding (Charles-Ogan, 2015), analyze data, develop algorithm, computation, modeling and simulation.

Researches that deal with ways to develop and support students' conceptual understanding in constructing mathematical knowledge in functions of several variables using educational software in general and MATLAB software in particular is succinct. Moreover, very few researches were done on software supported learning in combination with either collaborative learning approach or traditional lecture method in functions of several variables. Thus, this study, tried to investigate MATLAB supported learning and students' conceptual understanding in functions of several variables at Wolkite University.

MATLAB supported learning was chosen because of its applicability in wide areas of discipline like electrical engineering, mechanical engineering, computer science and so forth for the purpose of simulation work and program writing. On top of this, it is used as teaching and learning aid for mathematics students specially on sketching graphs of 2D and 3D in the Ethiopian context.

Conceptual understanding

Concept, understanding, mathematical understanding and conceptual understanding are very important terminologies in this study. Merriam-Webster dictionary defined concept as an idea of what something is or how it works; as a generic idea generalized from particular instance. Similarly, it defined understanding as a mental grasp; as knowledge and ability to judge a particular situation or subject. For Wiggins[1)] understanding means being able to justify procedures used or state why a process works.

Despite the meanings attributed to conceptual understanding, different literatures present conceptual understanding as conceptual knowledge (Rittle-Johnson & Schneider, 2015). Similarly, Star (2005) indicated that conceptual knowledge encompasses not only what is known but also the way concepts can be known. Conceptual knowledge might be considered as knowledge of the concept (Baroody, Feil, & Johnson, 2007). They argued that conceptual knowledge is about knowledge of facts, generalizations and principles.

According to Rittle-Johnson & Schneider (2015), understanding is an end product of conceptual knowledge. In his view conceptual understanding and conceptual knowledge are different. According to Hiebert (2003), understanding something is a network of concepts.

Significant numbers of researches have been reported on conceptual and procedural knowledge in mathematics. These two types of knowledge are assumed to be distinct yet related (Areaya & Sidelil, 2012). Different scholars define conceptual knowledge differently. For instance: Stump[2)] defined it as a knowledge that consists of rich relationships or web of ideas. For Schneider & Stern (2005) it is interplay of concepts and principles in a certain domain of knowledge. Similarly, Engelbrecht, Harding, & Potgieter (2005) defined it as the ability to form connection between concepts or between concepts and procedures. Whenever conceptual and procedural knowledge are raised in most literatures the name of two scholar Hiebert & Lefevre (1986) appears simultaneously. They define conceptual knowledge as knowledge of principles and relations between pieces of information in a certain domain, whereas procedural knowledge is the ability to quickly and efficiently solve problems.

Haapasalo & Kadijevich (2000) tried to redefine conceptual knowledge as having a dynamic nature. It is the ability to browse through networks consisting of concepts, rules, algorithms, procedures and even solve problems in various representational forms. Similarly, Grundmeier, Hansen, & Sousa (2006) pointed out that students prefer procedural knowledge over conceptual way of dealing with the problems in calculus in general and integration in particular whereas the study of Peteresson & Scheja (2008) showed that students developed their knowledge of integration in algorithmic way because it is more suitable for them and enable them to deal functionally and successfully with the presented tasks.

An equally important consideration to conceptual knowledge is procedural knowledge. Sometimes this knowledge could be defined as knowledge of rules and procedures to solve mathematical problems. It is also known as knowledge of operations (Schneider & Stern, 2010). According to Engelbrecht et al. (2005) it is the ability to solve problems through the manipulation of mathematical skills such as rules, formulas, algorithms and symbols in mathematics. Here, students follow steps to solve a given problem. Thus, procedural knowledge is the knowledge of how to solve a given problem using mathematical skills step by step in sequential order.

According to Rittle-Johnson & Siegler (1998) there is no fixed order in conceptual verses procedural knowledge. In some cases, a learner might acquire skill knowledge first whereas this might be reversed for the other cases where concept should come first. There are also some scholars who argued on the importance of both conceptual and procedural knowledge. According to Schneider & Stern (2005), teaching conceptual knowledge first leads to acquisition of procedural knowledge but the converse is not true.

In respective of the order of procedural and conceptual knowledge, misconceptions might happen. According to Roseman (1985) misconception obviously and frequently happens in mathematics due to errors made by students because of failure to understand key concepts. This indicates that students build and modify their existing conceptual understanding through involvement on construction process. The concept that a student construct through his/her own activities may differ from the formal mathematical concepts. This leads to cognitive conflict, which is the basis for further learning.

From the above definitions it is clear that learners use conceptual understanding to identify definitions and principles, what and when to use facts and principles, and to compare and contrast the relation between concepts. So, teachers are expected to teach learners develop conceptual understanding through posing problems that require students to reason, and make connection to what they already know using varieties of instructional method (NTCM, 2002).

Statement of the problem

According to Majid (2014), more than 75% of engineering courses offered are built on mathematical concepts. It is impossible to talk about science, engineering and technology without mathematics (Winkelman, 2009). Since, mathematics is considered a backbone for engineering (James & High, 2008) and it is a subject which seeks to understand the patterns that infuse both mind and world (Schoelfeld, 1992), it is evident that treating mathematical learning is of paramount importance. Contrary to this, many researchers indicated that engineering and technology students lack conceptual understanding (Goold, 2012; Huang, 2011; Majid, 2014).

One of the basic reasons is a lack of conceptual knowledge that occurs among students to learn concepts in mathematics (Handhika et al., 2016). Literature reveals that students have lack of conceptual understanding on some concepts of calculus of single and several variables (Martin-Blas et al., 2010; Martinez-Planell & Gaisman, 2013). Particularly, a research indicated that traditional method of teaching has shown inadequacy in developing a deep conceptual understanding. In the traditional method of teaching more focus is given to formula based problem solving rather than acknowledging the concepts behind (Antwi et al., 2011).

Despite these, there are few researches that were conducted on students' conceptual understanding of functions of several variables, particularly of functions of two variables (Dorko & Weber, 2014; Fisher, 2008; Kerrigan, 2015; Martinez- Planell & Gaisman, 2012; McGee & Moore-Russo, 2015; Tall, 1992). From these, it unraveled itself that there is a research gap on students' conceptual understanding on some concepts of functions of several variables like domain and range, limit and continuity, partial derivatives and multiple integrals of functions of several variables.

To scaffold students' conceptual understanding, many researchers recommended that feasible classroom instruction should be supported by technology for the sake of motivation (Melesse, 2014; Majid, 2014), visualization, and make the concepts clear and understandable (Idris, 2009). Literatures show that technology supported learning method in mathematics class enhances students' conceptual understanding (AlAmmary, 2012; Charles-Ogan, 2015).

Particularly, supporting instructional method with software gives student a privilege of "learning how to learn" through constructing their own understanding, and make the classroom environment attractive, interactive, and active as a cosmetic of teaching and learning process (Gemechu et al., 2013). Thus, this research was initiated to investigate MATLAB supported learning and student's conceptual understanding on concepts of functions of several variables.

General

The objective of this study was to investigate conceptual understanding of students when they learn through instructional approach supported with MATLAB and cooperative learning.

Specific objectives

The following are the specific objectives of the study: (1) to determine whether there is a significant mean difference between the mean scores of pre-test and post-test for the experimental groups and comparison group; (2) *T*o investigate the effect of MATLAB supported learning on students' conceptual understanding.

Research hypothesis

$H_{0[1]}$: There is no significant difference between the mean scores of the pre-test in conceptual understanding across all groups.

$H_{0[2]}$: There is no significant difference between the mean scores of the post-test of conceptual understanding across all groups.

Research question

The basic question is: what are the levels of first year engineering and technology students' conceptual understanding on functions of several variables? And do technology supported instruction effect students' understanding?

Research method and design

This study was conducted to investigate instructional approaches and their effect on conceptual understanding. Thus, a research method that fits for such kind is more of mixed research approach. One of the focuses of quantitative researches is testing a particular predetermined hypothesis through gathering numerical data whereas qualitative research gives more emphasis on understanding the phenomenon under investigation through bringing a word or picture data for thick description and interpretation (Tewksbury, 2009).

For this study, quantitative or qualitative research approach only could not address the problem at hand because there is a research question addressed only through qualitative research, particularly to explore level of students' conceptual understanding. Of the quantitative research approaches, non-equivalent pretest-posttest quasi experimental research design was employed. For qualitative part case study research design was used.

Sample and sampling techniques

In this study, four intact groups (Mechanical engineering group 1 and group 2, Garment engineering and Textile engineering) were involved where three of them were assigned as treatment groups and the remaining as a comparison group. All groups were selected through simple random sampling technique. The number of students involved in this study was 30, 29, 35 and 32 respectively.

Data collection tools

Two tiered conceptual tests (pre-test and post-test) were designed by the researchers. The intents of the instruments were to investigate conceptual understanding of students and the way they justify their answers. So, eight different questions were prepared beforehand and each question has four alternatives and reasoning part why they chosen a certain option. The face and content validity of the instruments were ensured through panel of experts working on the area. Based on the comments of those experts, necessary improvements were made and the revised tool was piloted before the actual administration to the students. The reliability of both tests was checked using Kuder-Richardson formula 20 (KR-20). The results of reliability coefficient of both tests were KR-20

= 0.6 for pre-test and 0.62 for post-test respectively which is in an acceptable level.

Procedures of data collection

Data were gathered through the research tools in four different phases.

Phase I: the researchers administered pre –test for all groups before any treatment was given to the experimental groups. The objective of giving the pre-test was to determine the basic knowledge level of all groups and to find out whether the previous knowledge of the students was homogeneous across all groups.

Phase II: General overview of MATLAB software training was given for the students under experimental groups in computer lab prepared for the purpose of this study in relation to functions of several variables for one week. The training was done by one of the researchers and the teachers who taught experimental groups using MATLAB.

Phase III: Two experimental groups were exposed to the lessons using MATLAB software supported learning method for two months whereas the third experimental group was taught through collaborative learning method only. At the same time, the comparison group was taught through traditional lecture method. All groups used the same textbook (i.e., Stewart, 2008) and covered the same materials in the course. All participant instructors had M.Sc. Degree and had more than 6 years teaching experience at university level who taught the course under investigation for many times.

Phase IV: All groups were exposed to post-test. This was done at the end of two months in order to determine which group was outperforming in conceptual understanding on the concepts of functions of several variables.

Method of data analysis

The data were found to be normal and assumptions of parametric statistics were not violated. In order to support or reject research hypotheses various descriptive and inferential statistics were employed. In addition, inferential statistics like ANOVA and ANCOVA were used to analyze the data at $\alpha = 0.05$ level. The effect size was also computed based on its significance. The qualitative data collected were analyzed through thematizing students' reasoning into four different concept areas like students' conception related to domain and range, limit and continuity, partial derivatives and their application and multiple integrals.

Results

The data is organized in tabular form and analyzed in line with the research hypotheses. The results and associated discussion are presented hereunder.

Quantitative results

Various descriptive and inferential statistical techniques were used according to their relevance to the data collected for this study. In this case demographic data, general descriptive statistics and inferential results of the data are presented as follows.

Demographic data

This study was conducted at Wolkite University, in Ethiopia located in Guraghe Zone, Southern Nations Nationalities and Peoples Region. From this university four intact classes were randomly selected and taken as samples of the study. These are: Mechanical engineering group 1, Mechanical engineering group 2, Garment engineering and Textile engineering departments. All students in those departments were considered as participants of the research.

All groups were exposed to different learning approaches in order to identify which learning approach is more effective to foster students' conceptual understanding. So, Mechanical engineering group 1 students learnt though MATLAB supported learning in combination with traditional lecture method, Mechanical engineering group 2 students learnt though MATLAB supported learning in combination with collaborative learning method, Garment engineering students learnt through collaborative method only and Textile engineering students learnt through the traditional lecture method.

The cut offline for achiever levels were done based of the students' previous Applied Mathematics grade report. Those of students scored A- and above were categorized under higher achievers, C+ to B+ were grouped under medium achievers and below C+ were grouped under low achievers.

Table 1. Demographic data of participants in those departments by sex and achiever levels

Sex	Achiever level	Department								Total	
		Mechanical 1		Mechanical 2		Textile		Garment			
		F	%	F	%	F	%	F	%	f	%
Male	Ha	7	23.33	7	24.14	9	28.13	2	5.714	25	19.84
	Ma	16	53.33	16	55.17	16	50	15	42.86	63	50
	La	4	13.33	3	10.34	3	9.375	3	8.571	13	10.32
	Total	27	90	26	89.66	28	87.5	20	57.14	101	80.19
Female	Ha	0	0	0	0	0	0	0	0	0	0
	Ma	3	10	3	10.34	3	9.375	7	20	16	12.7
	La	0	0	0	0	1	3.125	8	22.86	9	7.14
	Total	3	10	3	10.34	4	12.5	15	42.86	25	19.84
Total		30		29		32		35		126	

This demographic data reveals that female participants are few, but are equivalently present in each of the departments. Despite this, the numbers of students in the departments are more or less equivalent which would not violate measures of tendency and variation.

General descriptive statistics

The following presents the descriptive statistics of the students' conceptual understanding across all groups in their pre-test and post-tests.

The mean scores of the pretest and post-test conceptual understanding show that equivocally substantiated by a variation in their standard deviations. Though, in aggregate the averages are all above average of the scale (M = 3.00) some of the variations range up-to 1.85 which counter act to demand further analysis.

Table 2. Descriptive statistics of Pre-test and post-test for conceptual understanding across groups

Observations	Group	N	Mean	Std. Deviation	Std. Error
Pre-test for Conceptual Understanding	Mechanical 1	30	3.8333	1.23409	.22531
	Mechanical 2	29	3.9310	1.30742	.24278
	Garment	35	3.0286	1.42428	.24075
	Textile	32	3.2813	1.27594	.22556
	Total	126	3.4921	1.35497	.12071
Post-test Conceptual Understanding	Mechanical 1	30	4.3000	1.72507	.31495
	Mechanical 2	29	4.8966	1.31868	.24487
	Garment	35	3.8857	1.85934	.31429
	Textile	32	3.3750	1.43122	.25301
	Total	126	4.0873	1.68295	.14993

Equivalence of the groups at pre-test

Pre-test on conceptual understanding was conducted to measure the equivalence of the groups prior to the study or consider potential variations that could be considered as covariate for further inferential analysis. To do this, the researchers compared mean scores of pre-tests of all groups using ANOVA, as the Levene's Test was not significant for the pre-test conceptual understanding across all groups.

$H_{0[1]}$: There is no significant difference between the mean scores of the pre-test in conceptual understanding across all groups

The result of the ANOVA for the pre-test is presented in Table 3.

Table 3 shows the ANOVA values of F(3, 122) = 3.466 with P = 0.018 at P<0.05 for pretest scores of conceptual understanding. This result indicates that there is significant mean difference between the four groups on conceptual understanding before giving any treatments. This means the groups were not equally likely that demands filtering out of the pre-test as a covariate during post-test analysis.

Table 3. One-way analysis of variance summary table comparing all groups on pre-test of conceptual understanding

		Sum of Squares	df	Mean Square	F	P
Pretest	Between Groups	18.023	3	6.008	3.466	.018
	Within Groups	211.469	122	1.733		
	Total	229.492	125			

Equivalence of the groups at post-test

$H_{0[2]}$: There is no significant difference between the mean scores of the post-test of conceptual understanding across all groups.

Table 4. One-way analysis of variance summary table comparing all groups on post-test of conceptual understanding

		Sum of Squares	Df	Mean Square	F	P
Post-test	Between Groups	38.007	3	12.669	4.891	.003
	Within Groups	316.033	122	2.590		
	Total	354.040	125			

Table 4 above shows the value of F(3, 122) = 4.891 with p = 0.003 at p < 0.05 for post-test of conceptual understanding. This result indicates that there is significant mean difference between the four groups on conceptual understanding after treatments were given to experimental groups.

Since ANOVA test for post-test of conceptual understanding shows there is statistically significant difference between mean scores of each group, Post Hoc multiple comparison using Tukey HSD test was employed. The result of Tukey HSD shows that there is a significant mean difference on post-test of conceptual understanding between Mechanical group 2 students and the other group students at $P < 0.05$.

This is supported by different literature that indicates software integrated learning method in learning calculus class enhances students' conceptual understanding (AlAmmary, 2013; Charles-Ogan, 2015). Similarly, collaborative learning method has positive effect on students' conceptual understanding and problem solving (Wong, 2001).

As it was shown in the above table 3, the groups were not equivalent during pre-test. On top this, the post-test result shows that there is a significant difference on the mean score of students between Mechanical group 2 and Textile engineering students. This difference might be due to their prior difference since there is a significant difference on their pre-test result. So, in order to see effects of pretest as a covariate of conceptual understanding on post-test analysis of covariate (ANCOVA) was employed.

Table 5 below shows the adjusted and unadjusted mean scores of students' conceptual understanding during pre-test as a covariate respectively.

Table 5. Adjusted and Unadjusted groups means and variability for conceptual understanding using pre-test as a covariate

Groups	N	Unadjusted		Adjusted	
		M	SD	M	SE
Mechanical 1	30	4.3000	1.72507	4.242[a]	.295
Mechanical 2	29	4.8966	1.31868	4.822[a]	.301
Garment	35	3.8857	1.85934	3.965[a]	.275
Textile	32	3.3750	1.43122	3.411[a]	.284

a. Covariates appearing in the model are evaluated at the following values: Pretest = 3.4921.

The Estimated Marginal Means were statistically adjusted on the post-test mean scores for all groups to enable comparison between the pre-test and post-test, and among the groups in their post-test.

The mean scores of students' conceptual understanding across all groups are given above in Table 5 before and after controlling effect of pretest. The mean scores of post-test of conceptual understanding show that Mechanical group 2 students performed higher than others before and after controlling pre-test i.e. M = 4.8966 and (Mean adjusted) M_a = 4.822 respectively whereas Textile students were least in their post-test result i.e. M = 3.3750 and M_a = 3.411 respectively.

Table 6. Analysis of covariance for conceptual understanding as a function of groups, using pre-test as a covariate

Source	Mean Square	df	F	P	η^2
Pre-test	6.140	1	2.397	.124	.019
Post-test	10.065	3	3.930	.010	.089
Error	2.561	121			

a. R Squared = .125 (Adjusted R Squared = .096)
b. Computed using alpha = .05

Analysis of covariance was used to assess whether there is a statistically significant difference between groups on students' conceptual understanding after controlling the differences between each groups in pretest. The result indicated that after controlling the pre-test, there is a significant difference between groups on post-test of conceptual understanding, F(3,121) = 3.930, p= 0.01at p <0.05. The effect size after controlling pretest as covariate was represented by eta square η^2= 0.089 which is medium effect size for post-test of conceptual understanding between groups. On the other hand, for pretest of conceptual understanding the F values after adjusting the covariate was F(1,121) = 2.397, p= 0.124 at p <0.05. This indicates that there is no significant difference between adjusted pretest results on conceptual understanding across groups. Since analysis of covariance is significant for groups, Post hoc analysis was done using Bonferroni test. The result of Bonferroni shows that there is a significant mean difference on post-test of conceptual understanding between Mechanical group 2 students and Textile students ($p = 0.006$) on the adjusted mean. This difference comes not because of their prior difference, but because of intervention.

Qualitative result

The research question to be addressed through qualitative approach was: what are the levels of first year engineering and technology students' conceptual understanding on functions of several variables?

The observation made from students' reasoning indicated that they have problems of understanding the concept of domain and range in functions of two variables, and the way to write domain and range. This indicates that students did not develop new schema for functions of two variables.

Students were facing difficulty on the notion of functions of two variables. The researchers observed that $\mathbb{R}^2$and $\mathbb{R}^3$schemas are coordinated with $\mathbb{R}$ schema of functions of a single variable.

Students' conceptions from reasoning part are summarized into the following points:

Students' conceptions related to domain and range in functions of two variables

Function of two variables is a function whose domain is a subset of the plane $\mathbb{R}^2$ and range is a subset of R. If we denote the domain set by D, then a function f is a rule that assigns to every point $(x, y) \in D$ to a real number$f(x, y) \in R$. For function of three variables, every point $(x, y, z) \in B$ where $B \subseteq R^3$in assigned to a real number$f(x, y, z) \in R$.

With this definition, students were given a table of values where the values of x were in the first column and the values of y were in the first row of the table so that they compute the values of z = *f*(*x*, *y*). Students were asked to determine the domain from the table. 65 students out of 126 replied that domain of the function is the set containing all elements in the first column. This revealed that students had difficulty to extend domain of a function of a single variable to functions of several variables which has to be written in the form of an ordered pair. Only 43 students correctly replied that the domain of a function of two variables is the set of ordered pairs written as (x, y)that satisfies the function.

Some reasons given by students were as follows:

Domain is: (i) the first entry of the function; (ii) a number at which the given function is defined; (iii) a point at which the given function is undefined.

Range is: (iv) the value that we get by substituting all domain in the given function; (v) all values of *x* that make the given function different from zero; (vi) an output of a domain.

These indicate that students had difficulties in understanding domain and range of functions of several variables. According to Duval (2006) students who had difficulty of treatments and representations were categorized under the lower level on conception. The above reasons show that students under investigation were categorized under the lower level on conception before any treatment was given to them.

Students' conceptions related to limit and continuity of function of several variables

In a function of a single variable we say that a function $f(x)$ has a limit L at a point a if and only if for every $\varepsilon > 0$ there exists a positive number $\delta > 0$ such that $0 < |x - a| < a,\Rightarrow|f(x) - L| < \varepsilon$,for any x in the domain.

This definition can be similarly extended to functions of two variables. We say L is the limit of a function f of two variables at the point(a, b), written as $\lim_{(x,y)\to(a,b)} f(x, y) = L$ if for every $\varepsilon > 0$ there exists a positive number $\delta > 0$such that $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta \Rightarrow |f(x, y) - L| < \varepsilon$, for any (x, y) in the domain.

Despite the extension of the definition of a limit of function of a single variable to limit of a function of several variables, there is a main difference that worth understanding. The domain of a function of two variables is a subset of $\mathbb{R}^2$ which is a set of ordered pairs. So, the equivalent of $x \to a$ will be $(x, y) \to (a, b)$. For a function of three variables, the equivalent of $x \to a$ will be$(x, y, z) \to (a, b, c)$.

This has a very important consequence, one which makes computing limits for functions of several variables more difficult for students to easily grasp. While x could approach $a$in R from two directions, from the right and from the left, (x, y) can apporach (a, b) from infinitely many directions in $\mathbb{R}^2$. In fact, it does not even have to approach (a, b) along a straight path.

Students were asked to reason out what it means to say $\lim_{(x,y)\to(a,b)} f(x, y) = L$ exist. 75 students out of 126 replied that if $f(x, y) \to L$ no matter how $(x, y) \to (a, b)$ along any curve in the domain of f which is a correct reasoning. They were also asked to verify that a limit of a function at a point (a, b) exists and give their reasons. The reasons they gave were: some students reasoned out approaching (a, b) along a single curve in the domain, others reasoned along a single curve in the range and some others along any curve in the range of the function.

From these the researchers convict that students have observed difficulties such as gaps in: (a) conceptualizing why they consider different paths to in order to check the existence of limit; (b) converting algebraic expressions to graphical representations; (c) considering limit of function of two variables as the same as that of a function of a single variable; (d) some of them considered (a, b) as a subset of range of the function.

Students' conceptions related to partial derivatives and their applications

Derivatives of functions of a single variable represent the rate of change of the function with respect to changes in *x*. However, students had difficulty on extending this concept to derivatives in functions of several variables. They did not show difficulties of finding partial derivatives of functions of several variables, but they had difficulties in giving proper reason to its meaning. It was easy for students' to find partial derivatives of $f(x, y)$ with respect to x, and with respect to y keeping one of the variables constant.

On top of the above, students know that derivative of any constant function is zero. But, majority of the students had demonstrated difficulty in understanding the theorem that says "every differentiable function is continuous". Some students stated that "all continuous functions are differentiable". In trying out to reason an application of derivatives, some students tried to generalize that "if the first partial derivatives with respect to x and y of a function are greater than zero at a given point, then the function has maximum value" irrespective of the value of mixed derivative and the concept of saddle point in functions of several variables. This indicates that students had difficulty in determining extreme values and the relationship between continuity and partial derivatives, as well as partial derivatives and differentiability of a given function of several variables. Hence, these results revealed that majority of the students are categorized under the lower level on conceptions (Duval, 2006).

Students' conceptions related to multiple integrals

One of the conceptions of a definite integral $\int_a^b f(x)dx$ is an area of the plane region bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and

$x = b$. Similarly, double integral of a function of two variables over a domain D is viewed as a volume of the three dimensional region S bounded by the surface $z = f(x, y)$, the xy-plane, and the cylinder parallel to the z-axis passing through the boundaries of D, where D is the domain of integration. Double integral of $f(x, y)$ over the domain D is defined as $\iint_D^S f(x, y)\, dA$, in such a way, that its value will give the volume of the solid S, whenever D is a domain and f is its function with positive values.

In this regard, students were asked to give their reasons in sketching integrations, computing integrals and determining volumes.

To this end, students were given height, length and width of a parallelepiped to be $4cm, 3cm\ and\ 3cm$ respectively and they were asked to determine the algebraic representation where one of the vertices of the parallelepiped lies on the origin and its base is a square. Only 59 students correctly answered. Few students tried to sketch the graph on 3D. In addition to this, students were given a function $f(x, y)$ that is defined on a rectangle as shown in the figure below on R where $R = \{(x, y): a \leq x \leq b, c \leq y \leq d\}$. They were asked to determine the double integral in the region below $f(x, y)$ and above the xy-plane. 81 students replied correctly.

If $f(x, y) \leq 0$ on D, then students were required to determine the place where the volume (V) between the function and the region D exists. Majority of the students (61) replied that V is the volume of the solid lying vertically below D and above the surface $z = f(x, y)$. In this line, those who were found to have better reasons were those who were taught with the support of MATLAB. This was so, because MATLAB assisted them to physically visualize the region and the overall sketch of the integration. Despite these groups, many others had difficulties manifested with their work and reasons listed here: (i) they had difficulties on finding limits of multiple integrals; (ii) they had difficulty to reverse order of integration form horizontal region to vertical region and vice versa; (iii) lack of understanding of double integrals in functions of several variables as volume of the solid region bounded by the surface $z = f(x, y)$ and the xy-plane; (iv) they had difficulty to sketch graph and understanding of graph of solid regions, etc.

Discussion

The result shows that combination of MATLAB with collaborative method of instruction increases students' conceptual understanding when compared with collaborative method only and MATLAB with traditional method. This also agrees with the research work of Gemechu et al. (2013) that supporting instructional approach with educational software creates a privilege of learning how to learn through constructing their own understanding. On top of these, studies are indicating that supplementing instruction with educational technology helps teachers to develop students' conceptual understanding and problem solving skills (Almekhlafi & Almeqdadi, 2010; Jaun et al., 2012; Katehi, 2005). Albeit these, this study brought an insight to the use of software supported learning in combination with collaborative method. Studies also indicate that educational technology supported interdisciplinary collaborative learning and integration of mathematical and statistical software facilitates instruction of mathematics. For instance, hand held tools like calculators, mind tools like MATLAB, Mathematica, Maple, Fortran, C++ and so forth (Andreatos & Zagorianos, 2009; Charles-Ogan, 2015; Ogunkunle & Charles-Ogan, 2013) are useful in facilitating mathematics learning. Especially, MATLAB is used to visualize and plot different 2D and 3D graphs for better understanding and imagination of the problem (Charles-Ogan, 2015), analyze data, develop algorithm, computation, modeling and simulation. In spite of the advantages stated here, delivering instruction through MATLAB with collaborative learning further enhances conceptual understanding as manifested by the results of this study. Thus, the researchers are recommending the use of mathematical software supported with collaborative learning method in order to foster students' conceptual understanding.

The qualitative part of the study shows that students had difficulty in finding domain and range. This result is supported by Kashefi et al. (2010) that reveals students have difficulty in finding the range of functions of two variables. Students' response to the problems posed in pretest items show that they apply and use the concept of range of functions of a single variable to functions of several variables. After intervention, students' construction of range of function of several variables was improved through utilization of MATLAB supported learning in combination with collaborative learning method that helped them to understand that the range is the value of f on z $-$axis and they did not find it as an interval based on the graph of the domain in two dimensions.

The concepts in Applied Mathematics II are often an extension of the concepts in Applied Mathematics I. These require students to generalize the concepts in R^2 to a higher dimension. For instance Dorko & Weber (2014) shows that students generalized that if $f(x)$ has a domain in terms of x, then it is generalized that $f(x,y)$ has a domain in terms of x and y. Similarly, the range of single variable is usually associated with the vertical axis i.e. in terms of y $-$axis, so the range of $f(x,y)$ is the z $-$axis.

From the results of this study students' prior knowledge of domain and range, limit and continuity, derivatives and their application and integrals in functions of a single variable have impact on interpretation of the same in functions of several variables. However, researches depicted that students have some misconceptions such as the idea that in $f(x,y)$, domain is in terms of x and range is in terms of y (Dorko & Weber, 2014). Finding limit of $f(x,y)$ along a single curve was also another misconception that dragged the concept in a function of a single variable to functions of several variables. Students encounter epistemological problems with the limit concept and that these problems emanated from language and symbolism used. Robert & Speer (2001), Dreyfus

(1990), Eisenberg (1991), Orton (1983) all have shown that learners have difficulties in understanding limits and derivatives.

Except few students who learnt through the support of MATLAB in combination with collaborative methods, almost all other groups had the same problem of conceptual understanding on these concepts. The success of this approach is justified, because MATLAB offers them a room to visualize graphs, and overall properties of the graphs and the collaborative method gives them the room to discuss with their colleague on the concepts they feel challenged. This made them to perform better than the other groups. According to Kashefi et al. (2012) software supported learning helps to develop students' conceptual understanding of functions of two variables. However, this study shows that students learning through MATLAB supported learning in combination with collaborative method was the one that helped students perform better than other method of teaching. MATLAB assists learners develop better conceptual understanding as it demonstrates exemplary graphs which are helpful in characterizing the graph of a function and in identifying the region, and the limits of integration easily.

In the attempt to investigate students understanding some probing questions that require reversing the order of integration were given such as $\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dydx$. Few students replied that it is a must to reverse the order of integration since it is impossible to integrate the inner function as it is. Majority of them, however, replied that it is possible to integrate as it is. Some of them tried to justify the use of Fubini's theorem. These imply that students had difficulty on how to reverse order of integration and apply it. This seems a direct consequence of their knowledge of direct integration which caused them, to fail realizing the issue of reversing orders. Using MATLAB they could have seen the essence of reversing the order of integration and grasp the proper conception.

Conclusion

Although learning with the support of instructional technology was promoted to help develop a better understanding, integrating instructional technology with collaborative learning method was found to have accounted a comparatively better conceptual understanding.

It is thus; wise to consider the use of MATLAB integrated with collaborative learning method to enhance students' conceptual understanding. Therefore, universities need to take into account the use of this instructional method so that their students will have proper conceptual development and a better understanding of concepts of functions of several variables.

NOTES

1. https://grantwiggins.wordpress.com/2014/04/23/conceptual-understanding-in-mathematics/
2. https://web.pa.msu.edu/people/stump/stump

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