ESTIMATION OF POPULATION PROPORTION OF STIGMATIZED ATTRIBUTE USING BAYESIAN APPROACH

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Abstract. Bayesian Approach to Randomized Response Technique has been a technique for estimating the population proportion, especially of respondents possessing stigmatized attributes such as induced abortion, use of drugs and tax evasion. In this paper, we propose Bayesian estimators of population proportion of a stigmatized attribute assuming Kumaraswamy and the generalised beta prior using life data on induced abortion. The newly proposed Bayesian estimators were validated numerically for a large interval of the designed values of the population proportion at different sample sizes. It was observed that the newly developed Bayesian estimators were more sensitive in capturing stigmatized attribute than the Bayesian estimator developed by Hussain & Shabbir (2012) for relatively small, moderate as well as large sample sizes.

Keywords: Bayesian estimators, alternative priors, stigmatized attribute, mean square error, absolute bias

Introduction

Asking information about a stigmatized attribute such as induced abortion, use of drugs, and tax evasion in a human population is a complicated issue. Direct questioning approach generally leads to doctoring of the true responses. The reason may be fear of social stigma or counter attacks. But due to socioeconomic reasons, information about incidence of such attributes in the population becomes essential. Warner (1965) introduced a method of survey to gather information about stigmatized attributes by ensuring privacy to the respondents. Numerous developments and improvements on Warner's Randomized Response Technique have been put forward by many researchers. Greenberg et al. (1969), Folsom et al. (1973), Christofides (2003), Mangat (1994), Kim & Warde (2004), Adebola & Adepetun (2011), (2012a), (2012b), 2014) are some of the many to be mentioned.

At times, prior information about the unknown parameter may be available and can be used along with the sample information for the determination of that unknown parameter. This is called the Bayesian approach of estimation. The work on Bayesian analysis of randomized response techniques is not very much. However, attempts have been made on the Bayesian analysis of randomized response techniques. Winkler & Franklin (1979), Spurrier & Padgett (1980), O'Hagan (1987), Oh (1994), Migon & Tachibana (1997), Unnikrishnan & Kunte (1999), Barabesi & Marcheselli (2006; 2010), Hussain & Shabbir (2009a, 2009b; 2012), Hussain et al. 2010), Kim et al. (2006), Adepetun & Adewara (2014; 2015; 2016) are the major references on the Bayesian analysis of the Randomized Response Techniques.

This paper presents Bayesian analysis to Kim and Warde's Randomized Response Technique using alternative beta priors other than the simple beta prior used by Hussain & Shabbir (2012) in their published paper. The paper is arranged as follows: firstly, we present the methodology, then, the existing as well as the proposed alternative Bayesian estimation of population proportion, the next section contains the results and discussions, and finally is the conclusion.

Methodology

In Bayesian Analysis, the prior distribution or information about the unknown parameter of the population is combined with the sample information for the estimation of that unknown parameter. Notable authors like Winkler & Franklin (1979), O'Hagan (1987), Kim et al. (2006), Hussain & Shabbir (2009; 2012), Hussain et al. (2010) have provided Bayesian analysis to some randomized response techniques in the literature using simple beta distribution as their prior distribution.

In this research work, we presented both conventional and alternative beta priors for randomized response technique in the work. Similarly, we assume numerical values for the parameters in the priors.

In the case of simple beta prior, we assume $a > 1, b > 1, a \neq b, c = 1$. For Kumaraswamy prior, we assume $a = 1, b > 1, c > 1, b \neq c$. For the generalised beta prior, we assume $a > 1, b > 1, c > 1, a \neq b \neq c$ respectively. Consequently, the conventional estimator with simple beta prior along with the proposed estimators assuming Kumaraswamy and the generalised beta priors were derived and computed from their respective posterior distributions using R statistical software respectively.

The tables showing absolute bias and mean square errors were displayed for comparison using selected sample sizes 25,100, and 250 respectively.

Life data obtained from administered survey questionnaires on induced abortion among 300 women in Akure, Ondo State were also used to establish the efficiency of the proposed estimators in capturing responses from respondents with respect to stigmatized attribute.

Presentation of existing technique

Hussain & Shabbir (2012) in their referred paper presented a Bayesian analysis to the Randomized Response Technique proposed by Kim & Warde (2004) using a simple beta prior distribution to estimate the population proportion of respondents possessing stigmatized attribute.

Let the simple beta prior be defined as follows

$$f(\pi) = \frac{1}{\beta(a,b)}\pi^{a-1}(1-\pi)^{b-1} \ ; \ 0 < \pi < 1 \qquad (1)$$

where (a, b) are the shape parameters of the distribution and π is the population proportion of respondents possessing the stigmatized attribute. Let $X = \sum x_i$ denotes the total number of women who have committed abortion in a sample of size *n* drawn from the population. The conditional distribution of X given π was presented as

$$f(X|\pi) = \binom{n}{x}\phi^x(1-\phi)^{n-x} \qquad (2)$$

where $\phi = T\pi + 1 - T$ is the probability of "yes response" in a sample of size n. T and $1 - T$ are the predetermined probabilities respectively.

Then

$$f(X|\pi) = \binom{n}{x}T^n\sum_{j=0}^{x}\binom{x}{j}\pi^j d^{x-j}(1-\pi)^{n-x} \qquad (3)$$

where $d = \frac{1-T}{T}$

The joint probability density function of π and X was derived as follows

$$f(X,\pi) = \frac{\binom{n}{x}T^n}{\beta(a,b)}\sum_{j=0}^{x}\binom{x}{j} d^{x-j}\,\pi^{a-1+j}(1-\pi)^{n-x+b-1} \tag{4}$$

The marginal probability density function was derived as

$$f(X) = \binom{n}{x}\frac{T^n}{\beta(a,b)}\sum_{j=0}^{x}\binom{x}{j} d^{x-j}\beta(a+j,n-x+b) \tag{5}$$

Thus, the posterior distribution of π given X was given as

$$f(X|\pi) = \frac{\sum_{j=0}^{x}\binom{x}{j} d^{x-j}\,\pi^{a-1+j}(1-\pi)^{n-x+b-1}}{\sum_{j=0}^{x}\binom{x}{j} d^{x-j}\beta(a+j,n-x+b)} \tag{6}$$

Under the Square error loss, the Bayes estimator i.e the posterior mean was given as

$$\hat{\pi}_H = \frac{\sum_{j=0}^{x}\binom{x}{j} d^{x-j}\,\beta(a+j+1,n-x+b)}{\sum_{j=0}^{x}\binom{x}{j} d^{x-j}\beta(a+j,n-x+b)} \tag{7}$$

The bias as well as the mean square error of $\hat{\pi}_H$ corresponding to the sample of size *n* was given as

$$Bias(\hat{\pi}_H) = E(\hat{\pi}_H) - \pi \tag{8}$$

$$MSE(\hat{\pi}_H) = \sum_{x=0}^{n}(\hat{\pi}_H - \pi)^2\binom{n}{x}\phi^x(1-\phi)^{n-x} \tag{9}$$

Presentation of the proposed Bayesian techniques

In this section, we present a Bayesian analysis to Kim & Warde (2004) Randomized Response Technique assuming both Kumaraswamy and the generalised beta prior in addition to the simple beta prior used by Hussain & Shabbir (2012).

The Kumaraswamy prior distribution of π is given as

$$f(\pi) = bc\pi^{c-1}(1-\pi^c)^{b-1} \ ; b, c \ > 0 \tag{10}$$

The joint density function of π and X with Kumaraswamy prior is as follows

$$f(X,\pi) = bc\binom{n}{x}T^n\sum_{j=0}^{x}\binom{x}{j}\pi^j d^{x-j}(1-\pi)^{n-x}\,\pi^{c-1}(1-\pi^c)^{b-1} \tag{11}$$

The marginal probability density function is found by integrating (11) with respect to π as follows

$$f(X) = \binom{n}{x}T^n bc\sum_{j=0}^{x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{j}\binom{b-1}{k}d^{x-j}\beta(ck+c+j, n-x+1) \tag{12}$$

The posterior distribution is

$$f(\pi|X) = \frac{\sum_{j=0}^{x}\sum_{k=0}^{b-1}\binom{x}{j}\binom{b-1}{k}(-1)^k d^{x-j}\,\pi^{ck+c+j-1}(1-\pi)^{n-x}}{\sum_{j=0}^{x}\sum_{k=0}^{b-1}\binom{x}{j}\binom{b-1}{k}(-1)^k d^{x-j}\beta(ck+c+j, n-x+1)} \tag{13}$$

Thus, the posterior mean is

$$\hat{\pi}_{prop1} = \frac{\sum_{j=0}^{x}\sum_{k=0}^{b-1}\binom{x}{j}\binom{b-1}{k}(-1)^k d^{x-j}\,\beta(ck+c+j+1, n-x+1)}{\sum_{j=0}^{x}\sum_{k=0}^{b-1}\binom{x}{j}\binom{b-1}{k}(-1)^k d^{x-j}\beta(ck+c+j, n-x+1)} \quad (14)$$

The bias as well as mean square error of $\hat{\pi}_{prop1}$ is computed as

$$Bias(\hat{\pi}_{prop1}) = E(\hat{\pi}_{prop1}) - \pi \quad (15)$$

$$MSE(\hat{\pi}_{prop1}) = \sum_{x=0}^{n}(\hat{\pi}_{prop1} - \pi)^2 \binom{n}{x}\phi^x(1-\phi)^{n-x} \quad (16)$$

The generalised beta prior is defined as

$$(\pi) = \frac{c}{\beta(a,b)}\pi^{ac-1}(1-\pi^c)^{b-1}; \quad a, b, c > 0 \quad (17)$$

where $a, b,$ and c are the shape parameters of the prior distribution as given in Eq. (17).

The joint density function of π and X with the generalised beta prior is

$$f(X,\pi) = A\sum_{j=0}^{x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{j}\binom{b-1}{k}d^{x-j}\pi^{ac+j-1+ck}(1-\pi)^{n-x} \quad (18)$$

where $A = \frac{c}{\beta(a,b)}\binom{n}{x}T^n$

The marginal probability density function is

$$(X) = A\sum_{j=0}^{x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{j}\binom{b-1}{k}d^{x-j}\beta(ac+j+ck, n-x+1) \quad (19)$$

Thus, the posterior distribution of π given X is

$$f(\pi|X) = \frac{f(X,\pi)}{f(X)} = \frac{\sum_{j=0}^{x}\sum_{k=0}^{b-1}(-1)^k \binom{x}{j}\binom{b-1}{k} d^{x-j}\pi^{ac+j-1+ck}(1-\pi)^{n-x}}{\sum_{j=0}^{x}\sum_{k=0}^{b-1}(-1)^k \binom{x}{j}\binom{b-1}{k} d^{x-j}\beta(ac+j+ck, n-x+1)} \quad (20)$$

The posterior mean which is the Bayes estimator is derived as

$$\hat{\pi}_{prop2} = \frac{\sum_{j=0}^{x}\sum_{k=0}^{b-1}\binom{x}{j}\binom{b-1}{k}(-1)^k d^{x-j}\,\beta(ck+ac+j+1, n-x+1)}{\sum_{j=0}^{x}\sum_{k=0}^{b-1}\binom{x}{j}\binom{b-1}{k}(-1)^k d^{x-j}\beta(ck+ac+j, n-x+1)} \quad (21)$$

The bias of $\hat{\pi}_{prop2}$ is

$$Bias(\hat{\pi}_{prop2}) = E(\hat{\pi}_{prop2}) - \pi \quad (22)$$

The mean square error of $\hat{\pi}_{prop2}$ is

$$MSE(\hat{\pi}_{prop2}) = \sum_{x=0}^{n}(\hat{\pi}_{prop2} - \pi)^2 \binom{n}{x}\phi^x(1-\phi)^{n-x} \quad (23)$$

Results and discussions

We wrote suitable codes using R-statistical software to evaluate the derived estimators, bias and mean square errors which are given by equations 8, 9, 14, 15, 20, and 21 at sample sizes 25, 100, and 250 respectively. From the results presented in tables 1a to 6b respectively, when $n = 25, T = 0.1$ and 0.2, the conventional simple beta estimator is better than the proposed Bayesian estimators when π lies within the range of $0.1 \leq \pi < 0.2$ while the proposed Bayesian estimators are better than the conventional simple beta estimator when π lies within the range of $0.2 < \pi < 1$. However, the proposed Bayesian estimator which assumes the generalised beta prior is the best in obtaining more responses from respondents when π lies within the range of $0.3 < \pi < 1$. When

$n = 100, 250$; $T = 0.1$ and 0.2, the proposed Bayesian estimators are better than the conventional simple beta estimator when π lies within the range of $0.1 < \pi < 1$. However, the proposed Bayesian estimator which assumes the generalised beta prior is the best in obtaining more responses from respondents when π lies within the range of $0.1 < \pi < 1$ respectively.

Table 1a. Mean square errors for Kim & Warde (2004) RRT at $n = 25, x = 11, T = 0.1$

π	MSEBETA	MSEKUMA	MSE GLS
0.1	4.470146E-12	5.203827E-10	2.395927E-09
0.2	2.815330E-10	1.535606E-11	9.033024E-10
0.3	1.272503E-09	2.242367E-10	1.245852E-10
0.4	2.977381E-09	1.147025E-09	5.977522E-11
0.5	5.396165E-09	2.783720E-09	7.088725E-10
0.6	8.528857E-09	5.134322E-09	2.071877E-09
0.7	1.237546E-08	8.198832E-09	4.148789E-09
0.8	1.693596E-08	1.197725E-08	6.939608E-09
0.9	2.221038E-08	1.646957E-08	1.044433E-08

Table 1b. Absolute bias for Kim & Warde (2004) RRT at $n = 25, x = 11,\ T = 0.1$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.01119064	0.12074121	0.25907821
0.2	0.08880936	0.02074121	0.15907821
0.3	0.18880936	0.07925879	0.05907821
0.4	0.28880936	0.17925879	0.04092179
0.5	0.38880936	0.27925879	0.14092179
0.6	0.48880936	0.37925879	0.24092179
0.7	0.58880936	0.47925879	0.34092179
0.8	0.68880936	0.57925879	0.44092179
0.9	0.78880936	0.67925879	0.54092179

Table 2a. Mean square errors for Kim and Warde (2004) RRT at $n = 25, x = 11, T = 0.2$

π	MSEBETA	MSEKUMA	MSE GLS
0.1	1.298247E-11	6.368725E-10	2.645818E-09
0.2	2.337871E-10	4.023513E-11	1.059132E-09
0.3	1.168499E-09	1.575050E-10	1.863521E-10
0.4	2.817118E-09	9.886822E-10	2.747996E-11
0.5	5.179645E-09	2.533767E-09	5.825151E-10
0.6	8.256078E-09	4.792758E-09	1.851458E-09
0.7	1.204642E-08	7.765657E-09	3.834307E-09
0.8	1.655067E-08	1.145246E-08	6.531064E-09
0.9	2.176882E-08	1.585318E-08	9.941729E-09

Table 2b. Absolute bias for Kim & Warde (2004) RRT at $n = 25, x = 11, T = 0.2$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.24462626	0.80499236	0.84070996
0.2	0.14462626	0.70499236	0.74070996
0.3	0.04462626	0.60499236	0.64070996
0.4	0.05537374	0.50499236	0.54070996
0.5	0.15537374	0.40499236	0.44070996
0.6	0.25537374	0.30499236	0.34070996
0.7	0.35537374	0.20499236	0.24070996
0.8	0.45537374	0.10499236	0.14070996
0.9	0.55537374	0.00499236	0.04070996

Comment: When $n = 25, T = 0.1$ and 0.2, the conventional simple beta estimator is better than the proposed estimators when π lies within the range of $0.1 \leq \pi < 0.2$ while the proposed estimators are better than the conventional simple beta estimator when π lies within the range of $0.2 < \pi < 1$. However, the proposed estimator which assumes the generalised beta prior is the best in obtaining more responses from respondents when π lies within the range of $0.3 < \pi < 1$ respectively.

Table 3a. Mean square errors for Kim & Warde (2004) RRT
at $n = 100, x = 43, T = 0.1$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	8.967545E-33	1.979483E-33	1.785698E-33
0.2	5.755682E-32	3.598771E-32	1.059981E-32
0.3	1.483197E-31	1.121695E-31	6.158749E-32
0.4	2.812561E-31	2.305249E-31	1.547487E-31
0.5	4.563660E-31	3.910538E-31	2.900836E-31
0.6	6.736496E-31	5.937563E-31	4.675919E-31
0.7	9.331067E-31	8.386324E-31	6.872739E-31
0.8	1.234737E-30	1.125682E-30	9.491294E-31
0.9	1.578542E-30	1.454905E-30	1.253159E-30

Table 3b. Absolute bias for Kim & Warde (2004) RRT
at $n = 100, x = 43,\ T = 0.1$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.06521261	0.03063872	0.02910039
0.2	0.16521261	0.13063872	0.07089961
0.3	0.26521261	0.23063872	0.17089961
0.4	0.36521261	0.33063872	0.27089961
0.5	0.46521261	0.43063872	0.37089961
0.6	0.56521261	0.53063872	0.47089961
0.7	0.66521261	0.63063872	0.57089961
0.8	0.76521261	0.73063872	0.67089961
0.9	0.86521261	0.83063872	0.77089961

Table 4a. Mean square errors for Kim & Warde (2004) RRT
at $n = 100, x = 43, T = 0.2$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	7.984359E-33	1.190816E-33	3.334982E-33
0.2	5.502221E-32	3.229967E-32	7.649880E-33
0.3	1.442336E-31	1.055821E-31	5.413835E-32
0.4	2.756186E-31	2.210381E-31	1.428004E-31
0.5	4.491772E-31	3.786677E-31	2.736360E-31
0.6	6.649093E-31	5.784708E-31	4.466452E-31
0.7	9.228150E-31	8.204475E-31	6.618279E-31
0.8	1.222894E-30	1.104598E-30	9.191842E-31
0.9	1.565147E-30	1.430922E-30	1.218714E-30

Table 4b. Absolute bias for Kim & Warde (2004) RRT
at $n = 100, x = 43,\ T = 0.2$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.06153396	0.02376387	0.03976872
0.2	0.16153396	0.12376387	0.06023128
0.3	0.26153396	0.22376387	0.16023128
0.4	0.36153396	0.32376387	0.26023128
0.5	0.46153396	0.42376387	0.36023128
0.6	0.56153396	0.52376387	0.46023128
0.7	0.66153396	0.62376387	0.56023128
0.8	0.76153396	0.72376387	0.66023128
0.9	0.86153396	0.82376387	0.76023128

Table 5a. Mean square errors for Kim & Warde (2004) RRT
at $n = 250, x = 106\ , T = 0.1$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	7.329826E-77	5.052631E-77	1.914836E-77
0.2	3.452198E-76	2.933285E-76	2.072026E-76
0.3	8.178880E-76	7.368774E-76	5.960036E-76
0.4	1.491303E-75	1.381173E-75	1.185551E-75
0.5	2.365464E-75	2.226215E-75	1.975846E-75
0.6	3.440373E-75	3.272004E-75	2.966887E-75
0.7	4.716028E-75	4.518540E-75	4.158674E-75
0.8	6.192429E-75	5.965822E-75	5.551209E-75
0.9	7.869578E-75	7.613852E-75	7.144490E-75

Table 5b. Absolute bias for Kim & Warde (2004) RRT
at $n = 250, x = 106\ , T = 0.1$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.08545503	0.07094954	0.04367738
0.2	0.18545503	0.17094954	0.14367738
0.3	0.28545503	0.27094954	0.24367738
0.4	0.38545503	0.37094954	0.34367738
0.5	0.48545503	0.47094954	0.44367738
0.6	0.58545503	0.57094954	0.54367738
0.7	0.68545503	0.67094954	0.64367738
0.8	0.78545503	0.77094954	0.74367738
0.9	0.88545503	0.87094954	0.84367738

Table 6a. Mean square errors for Kim & Warde (2004) RRT at $n = 250, x = 106, T = 0.2$

π	MSE BETA	MSEKUMA	MSE GLS
0.1	7.045008E-77	4.599335E-77	1.439075E-77
0.2	3.390056E-76	2.822565E-76	1.907759E-76
0.3	8.083078E-76	7.192663E-76	5.679077E-76
0.4	1.478357E-75	1.357023E-75	1.145786E-75
0.5	2.349152E-75	2.195526E-75	1.924411E-75
0.6	3.420695E-75	3.234776E-75	2.903783E-75
0.7	4.692984E-75	4.474772E-75	4.083902E-75
0.8	6.166019E-75	5.915516E-75	5.464767E-75
0.9	7.839802E-75	7.557006E-75	7.046379E-75

Table 6b. Absolute bias for Kim & Warde (2004) RRT at $n = 250, x = 106, T = 0.2$

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.08377830	0.06769215	0.03786452
0.2	0.18377830	0.16769215	0.13786452
0.3	0.28377830	0.26769215	0.23786452
0.4	0.38377830	0.36769215	0.33786452
0.5	0.48377830	0.46769215	0.43786452
0.6	0.58377830	0.56769215	0.53786452
0.7	0.68377830	0.66769215	0.63786452
0.8	0.78377830	0.76769215	0.73786452
0.9	0.88377830	0.86769215	0.83786452

Comment: When $n = 100, 250;\ T = 0.1$ and 0.2, the proposed estimators are better than the conventional simple beta estimator when π lies within the range of $0.1 < \pi < 1$. However, the proposed estimator which assumes the generalised beta prior is the best in obtaining more responses from respondents when π lies within the range of $0.1 < \pi < 1$ respectively.

Conclusion

We have proposed alternative Bayesian estimators of population proportion when life data were gathered through the administration of questionnaires on abortion on 300 matured women in addition to the conventional simple beta estimator proposed by Hussain & Shabbir (2012). We observed clearly from the results presented in tables and figures above, that for relatively small, intermediate as well as large sample sizes, the proposed Bayesian estimators are more sensitive in capturing sensitive attribute than the conventional simple beta estimator. However, the proposed generalised beta estimator is the best in obtaining information from respondents in survey which asks sensitive questions.

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