AVAILABILITY OF RESOURCES, MATHEMATICS TEACHERS' KNOWLEDGE, AND ATTITUDE TOWARDS MATHEMATICS VISUALIZATION AS PREDICTORS OF THE DEVELOPMENT OF STUDENTS' VISUALIZATION IN MATHEMATICS

Malugeta ATNAFU, Demis ZERGAW
Addis Ababa University, ETHIOPIA

Abstract. The purpose of this study was to assess the contributions of availability of resources, Mathematics teachers' knowledge and attitude towards teaching Mathematics using visualization techniques to developing students' visualization of Mathematics. The study used survey design with quantitative research method, and the instrument was a Likert scale questionnaire. Data were analysed employing mean, standard deviation, correlation, and multiple regression.183 teachers were selected using stratified random sampling from different Mathematics programs. Results indicated that Mathematics teachers suffered from shortage of resources to implement visualization techniques in teaching Mathematics. The knowledge teachers had to use Mathematics visualization techniques was below the expected level. Their attitude towards teaching Mathematics using visualization techniques, usefulness of Mathematics visualization techniques; and enjoying using Mathematics visualization techniques in teaching Mathematics were positive and above the expected level. However, their confidence in applying Mathematics visualization techniques in teaching Mathematics was below the expected level. The frequency of practice of Mathematics teachers to develop students' visualization in Mathematics was below the expected which means it was not implemented adequately. The availability of resources for using mathematical visualization techniques was the most dominant predictor variable for developing students' mathematical visualization. The next significant predictor was Mathematics teachers' knowledge to use Mathematics visualization techniques. However, Mathematics teachers' attitude towards using Mathematics visualization techniques contributed insignificantly to the variation in the development of students' Mathematics visualization abilities. Based on the outcomes of the study, it was recommended that schools should provide Mathematics visualization resources and training should be given to teachers on how to apply Mathematics visualization techniques. Besides, curriculum and textbooks should be designed in ways that facilitate the application of visualization in the teaching-learning of Mathematics.

Keywords: visualization, knowledge, attitude, resource, mathematics

---

Introduction

Mathematics is nowadays used as a tool for solving livelihood problems since it forms the basis for the application of science and technology. It is difficult for science and technology to solve problems without the application of basic Mathematical knowledge. Due primarily to this immense role played by Mathematics knowledge, the subject is offered in the entire system of education of Ethiopia ranging from primary through to the university. However, because Mathematics is more an abstract field, it is not adequately applied to daily life which is one of the reasons for low level of student achievement (Sezer, 2010).

One mechanism for dealing with these problems is using visualization techniques.

Visualization techniques help teachers and students understand mathematical concepts and solve problems. These techniques include concept map (Anderson-Inman & Ditson, 1999); graphs and pictorial presentations (Cifuentes & Hsieh, 2003); real-life applications;[1)] examples, non-examples and counter examples (Yanuarto, 2016); compare and contrast (Marzano et al., 2001) and experimentations (Michelsen, 2006); manipulatives such as real objects, models and paper-folding (Raphael & Wahlstrom, 1989); and computer applications such as animation and simulation (Michael, 2002).

Resources for visualization techniques

Educational researchers have recently begun to focus on visualization tools in order to enhance student learning at all levels (Stieff et al., 2007). With the rapid advancement of technologies, representation of concepts became virtual. Besides, a wide range of visualization tools is now accessible to teachers and students to visualize experimental data sets and simulate experiments or construct models of imperceptible entities (Stieff et al., 2007). For Rapp (2007), these tools have three characteristics that make them very useful and appealing: they are engaging; are interactive allowing students to manipulate variables; and are instrumental for improving learning through conveying information in a succinct, guided manner and that which aligns with the nature of mental representations. One key aspect of teachers' role in planning and managing learning is the skill to select the right resources for the right teaching and learning objectives (Wellington, 2002). ICT and multimedia tools are complex resources that can provide a range of possibilities for better learning of sciences including Mathematics (Webb, 2005). As in Kozma (2000), students' discourses and social interactions are influenced by the resources available to them.

Mathematics teachers' knowledge on visualization techniques

Linn (2003) indicated that teachers need extensive background knowledge in order to interpret visualizations. For the purpose of integrating visualization tools and make Mathematics comprehensible to students, teachers need to apply their pedagogical content knowledge in addition to knowledge of curriculum, learners, and educational context, etc (Webb, 2010). Besides, teachers need to have knowledge and skills of how to use a wide range of available technologies to support the content to be taught and the pedagogical approaches to fit the purposes.

Osborne & Hennessy (2003) stressed that many teachers employ ICT after teaching several lessons while the process of learning should move into the computer world very fast. It is a challenge for most teachers to use computational tools in lessons. Technologies change much faster than teachers manage to use them. And, often teachers are in a hurry to prepare students to the exam and cannot spend more time to learn technologies themselves and to prepare students to use them (Jasute, 2013).

Mathematics teachers' attitude towards visualization techniques

Mayer (2001) claimed that teachers demonstrated a range of positive responses for using science visualization programs. Further, it was indicated[2] that, teachers had a positive attitude towards visualization tools. Yet, there are several barriers to the successful integration of visualization techniques in teaching and learning environments. Becta[3] divided the barriers into two categories namely: (1) teacher-level barriers (individual), such as lack of time, lack of confidence, and resistance to change; (2) school-level barriers (institutional), such as lack of access to resources.

Teacher lack of confidence is directly related to the lack of competence in integrating ICT into pedagogical practice.[3]

Developing students' mathematics visualization

Visualization is the ability to see and understand a problem situation in mind. Visualizing a situation or an object involves "mentally manipulating various alternatives for solving a problem related to a situation or object without the benefit of using concrete manipulatives" (MOE, 2001). Visualization can be a powerful cognitive tool in problem solving. In the revised "Primary Mathematics Syllabus of Ethiopia" (MOE, 2007), it is highlighted as an important skill "essential in the learning and application of Mathematics". This ability to reason visually is increasingly important in the information age, and thus, the role that visualization plays in students' mathematical thinking and problem-solving experiences has become more significant (MOE, 2007).

To help students develop visualization skills, classroom teachers and designers of curriculum materials should first be aware of the factors that influence students' choice of problem-solving method, and of the processes and roles that visualization plays in mathematical problem solving (Yin, 2009). According to Yin (2009), both teachers and students see the role of visualization clearly and use it to help them in their problem-solving process. Yin (2009) also recommends that teachers increase students' awareness of unusable diagrams by illustrating the disadvantages of using such diagrams during problem solving.

The four steps for developing students' Mathematics visualization are: (i) designing lessons using visualization techniques; (ii) teaching mathematical concepts using visualization techniques; (iii) exposing students to practice the visualization of mathematical concepts; (iv) assessing students' progress on visualizing mathematical concepts.

Designing lessons using visualization techniques

Linn (2003) found that visualizations are useful for interpreting ideas. Linn (2003) indicated that learners may be confused by scientific visualizations because they do not have the same background knowledge as the people who created the visualizations. Although Linn (2003) recognized the role of technology in science, she concluded that "The appeal of visualizations overshadows the challenges of designing effective material". Linn's (2003) concerns draw attention to the importance of planning when and how to use different types of visualization in order to maximize their usefulness. As Tufte (1990) made clear, the design of graphic representation is important. To be effective, visual representations must first be well designed.

Teaching mathematical concepts using visualization techniques

Linn (2003) explained that instruction is important to ensure effective use of visualizations in science; and, without instruction in visualization techniques, students face difficulties in interpreting three-dimensional information. Linn (2003) proposed that combined with Problem-based Learning, visualization uses classroom technology to help learners create a visual picture of concepts and make connections between Mathematics and science. According to Bransford et al. (1999), technology can be used to help provide five key conditions for learning including: (i) real-world contexts for learning; (ii) connections to outside experts; (iii) visualization and analysis tools; (iv) scaffolds for problem solving; (v) opportunities for feedback, reflection and revision.

There is an important geometrical structure used to teach Mathematics which is called semi-concrete structure. An important component of forming concrete or, at least, semi-concrete mental representation of a concept is an external or physical reference (Konyalioglu et al. 2003). It is an effective use of semi-concrete structures that helps to teach abstract concepts in Mathematics. Graph, diagram, picture and geometrical shape or models are tools for visualization of abstract concepts in Mathematics. Through these, human reason sets up a relation between physical or external world and abstract concepts (Konyalioglu 2003).

Exposing students to practice the visualization of mathematical concepts

Practising visualization techniques is important for student learning. For example, Rieber (1990) showed that animated presentations of contents of lessons influenced student performance when practice was provided; however, this effect was eliminated without practice. The image of mathematical object is strictly linked to the image of the mathematical activity that is negotiated by learners in school practice. Taking into account school practice in which students and teachers are involved, the relationships between external visual representation, mathematical discourse and mental images can be sketched in this way (Phillips et al., 2010).

Assessing students' progress on visualizing mathematical concepts

The success of educational visualization depends on what learners bring to the task in terms of background knowledge, visio-spatial skills and interpretive ability. As a result, it is critically important to have a thorough understanding of the nature of visualization objects, their functions and the interpretive skills essential to assess the plausibility, validity, and value of visual images (Phillips et al., 2010). Teaching students how to work with visualization objects, monitoring and assessing the appropriateness and effectiveness of visualizations is important (Cifuentes & Hsieh 2003).

This study used the four steps stated above in order to investigate the extent to which teachers use them to develop student visualizations in Mathematics. And, in order to achieve this, the model indicated below was used.

In this model, the relationship between each of the independent variables such as availability of resources with the dependent variables such as designing lessons using visualization techniques is indicated. In addition to that, the relationships of each of the variables to one another and the development of student visualization in Mathematics are indicate

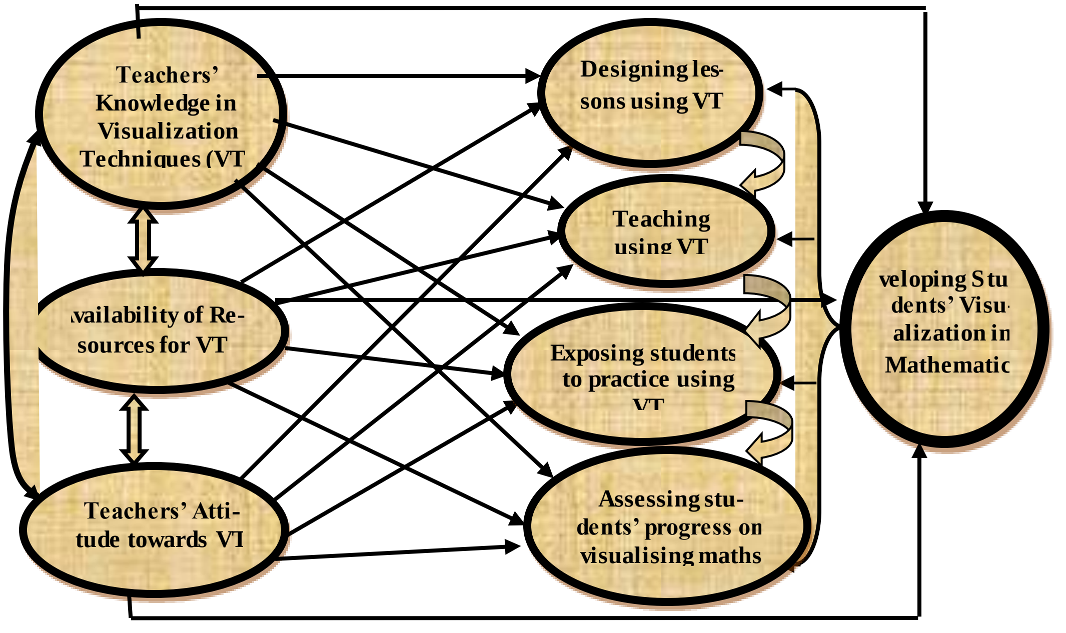

Figure 1. Conceptual framework of the study

Statement of the problem

Traditional Mathematics teaching mainly cultivates skills neglecting conceptual understanding of the underlying domain. Student learning difficulties in acquiring the concepts of Mathematics is more related to the abstract nature of Mathematics. Since mathematical concepts are abstract, students learn them through memorization. One of the most important problems associated with the teaching of Mathematics is related to students' difficulties in establishing relationship between their knowledge and intuition about concrete structures and abstract nature of Mathematics (Kadijevic, 1999).

In Mathematics learning, visualization can be a powerful tool of exploring problems and giving meaning to concepts and the relationship among them. Such a practice allows reducing complexity when dealing with a multitude of information (Rösken & Rolka, 2006). Use of the visualization approach provides students with opportunities to look at Mathematics as an accumulation of abstract structures and concepts from a different perspective. Wu & Shah (2004) highlight the importance of learner differences and the role of visualization in reducing how much students have to remember. They claim that visualizations provide multiple representations and descriptions of the same information, which enables students to visualize the connections between representations and relevant concepts. Visual representations have several important functions: (a) making connections visible; (b) presenting the dynamic and interactive nature of the subject; (c) promoting transformation from two-dimensional and three-dimensional thinking: (d) reducing how much students need to remember by making information explicit.

Therefore, as indicated above, the purpose of this study was to assess the contributions of availability of resources, Mathematics teachers' knowledge and attitude towards teaching Mathematics using visualization techniques on developing students' visualization in Mathematics.

Research questions

The research questions for the study are: (1) what are the levels of the availability of resources (AR), Mathematics teachers' knowledge (MTK) and attitude towards teaching Mathematics using visualization techniques (MTA), and developing students' Mathematics visualization (MTDSMV); (2) are there relationships between the availability of resources, Mathematics teachers' knowledge and attitude towards teaching Mathematics using visualization techniques, and developing students' Mathematics visualization; (3) what are the contributions of availability of resources, Mathematics teachers' knowledge and attitude towards teaching Mathematics using visualization techniques to developing students' Mathematics visualization.

Material and methods

Research design

This study used survey design with quantitative research method and employed a Likert scale questionnaire.

Population and sampling techniques

The population for this study consisted of all in-service Mathematics teachers of the Addis Ababa University who were pursuing their study during the summer of 2016. Out of a total of 524 in-service Mathematics teachers, 183 teachers were selected using stratified random sampling technique. The basis of stratification was the programs which the trainees pursued namely undergraduate, Post Graduate Diploma in Teaching (PGDT) and graduate.

Instruments of data collection

A Likert scale questionnaire was developed by adapting from Alias (2000) and Alias et al. (2002). The four components of the scale and the items each has are indicated below: (a) 15 items for availability of resources to teach mathematical visualization techniques; (b) 14 items for Mathematics teachers' knowledge on Mathematics visualization techniques. (c) 32 items for the attitude of Mathematics teachers towards teaching Mathematics using visualization techniques which was divided into confidence in applying visualization techniques (10 items), usefulness of Mathematics visualization techniques (12 items), and enjoyment in using Mathematics visualization techniques (10 items); (d) 34 items for developing students' visualization in Mathematics which was divided into four major components namely designing lessons using visualization techniques; teaching mathematical concepts using visualization techniques; exposing students to practise visualization of mathematical concepts and assessing students' progress on visualizing mathematical concepts.

All the scales for Mathematics teachers' knowledge and attitude towards Mathematics visualization techniques employed a five points scale ranging from

'Strongly agree' to 'Strongly disagree'. On the other hand, the instrument for availability of resources for teaching Mathematics visualization techniques had a five-point alternatives ranging from 'Not available at all' to 'Available to a great extent' while the instrument for the frequency of developing students' visualization of Mathematics concepts included a five-point alternative ranging from 'Not at all' to 'Always'**.**

Validity and reliability of the instrument

The instrument developed to measure the variables stated above was reviewed and amended based on the comments of professionals for its face and content validity. It was, then, pilot tested with 35 randomly selected in-service Mathematics teachers who were not included in the main study. The pilot study yielded the following Cronbach alpha coefficient values for its different components: (i) 0.884 for the availability of resources for teaching mathematical visualization techniques; (ii) 0.802 for Mathematics teachers' knowledge on Mathematics visualization techniques; (iii) 0.812 for the attitude of Mathematics teachers towards teaching Mathematics using visualization techniques; (iv) 0.703 for the confidence in applying visualization techniques; (v) 0.871 for the usefulness of Mathematics visualization techniques; (vi) 0.724 for enjoyment in using Mathematics visualization techniques; (vii) 0.891 for developing students' visualization in Mathematics; (viii) 0.716 for designing lessons using visualization techniques; (ix) 0.785 for teaching mathematical concepts using visualization techniques; (x) 0.782 for exposing students to practice visualization of mathematical concepts; (xi) 0.774 for assessing students' progress on visualizing mathematical concepts.

The afore-mentioned coefficients indicated that the values have acceptable internal-consistency values and, thus, are reliable.

Procedure of data collection

The instrument was administered to the participants during their regular class time with permission from two departments namely the Department of Mathematics and the Department of Science & Mathematics Education of the Addis Ababa University. The instructors of the students assisted in the data collection following orientation on how to administer the instrument. Data were collected on the spot after all the participants confirmed consent to take part in the study.

Method of analysis

Since the instrument was an ordinal five-point Likert scale and the skewness of the distribution for all the items and each component lied between -1 and +1, it was possible to take the data as one not significantly different from normal. Such an outcome also indicated that the variable is distributed approximately normally and allows using parametric tests. Therefore, the data analysis techniques used for this study were Mean, Standard deviation, One-sample t-test, Correlation, and Multiple regression. Before conducting the analysis on the data, all the assumptions of One-sample t-test, Correlation, and Multiple regression were checked.

Results

The first research question was related to the levels of the availability of resources, Mathematics teachers' knowledge and attitude towards teaching Mathematics using visualization techniques, and developing students' Mathematics visualization. Table 1 presents descriptive statistics and One-Sample t-test on the availability of resources for teaching visualization techniques.

Table 1. Mean, SD and One-Sample t-values on the Availability of resources (AR) for Teaching Mathematical Visualization Techniques (VTs), (N = 183)

No	Variables	Mean	SD	t
1	My school has reference books to support my teaching using VTs	2.93	1.20	-12.12*
2	I have reference books to support my teaching using VTs	3.18	1.22	-9.06*
3	My school has colour chalk to draw graphs and pictures	3.11	1.42	-8.44*
4	My school has real objects to clarify mathematical concepts	2.65	1.22	-14.95*
5	I have real objects to clarify mathematical concepts	2.78	1.19	-13.91*
6	My school has models/kites to clarify mathematical concepts	2.72	1.23	-14.18*
7	My school has scissors, papers, and rulers to prepare models/kites	3.28	1.17	-8.28*
8	I have scissors, papers, and rulers to prepare models/kites	3.05	1.23	-10.43*
9	My students have scissors, papers, and rulers to prepare models/kites	2.56	1.23	-15.82*
10	My school has computers to support my teaching using VTs	2.13	1.32	-19.16*
11	My school has internet access for me to support my teaching using VTs	2.04	1.32	-20.10*
12	My school has internet access for students to support their learning	1.83	1.22	-24.02*
13	My school has mathematical software for my teaching using VTs	1.53	1.06	-31.44*
14	I have mathematical software for my teaching using VTs	1.65	1.10	-28.94*
15	My school has mathematical videos for my teaching using VTs	1.56	1.13	-29.20*
	Availability of resources for teaching mathematical VTs	**2.47**	**.83**	-25.02*

* p< 0.5

* Note: The observed means of Availability of Resources (AR) items were compared against a specified mean (Test value) = 4 (Available)

Table 1 indicates that the aggregate average value of respondents on the availability of resources for teaching visualization techniques was significantly below the specified mean 4 (Available) (M=2.47, SD= .83, t =-25.02, p <.05). The result could be attributed to acute shortage of

- computers in schools (M=2.13, SD= 1.32, t = -19.16, p <.05),
- mathematical software (M = 1.53, SD = 1.06, t = -31.44, p <.05),
- mathematical videos (M = 1.56, SD = 1.13, t = -29.20, p < .05),
- internet access for teachers (M = 2.04, SD = 1.32, t = -20.10, p< .05) and

- internet access for students (M = 1.83, SD = 1.22, t= -24.02, p< .05); and
- mathematical software for teachers (M = 1.65, SD =1.10, t = -28.94, p<.05)

There was also shortage of real objects or models/kits, and students didn't have scissors, papers, and rulers to prepare models/kits to practise mathematical properties using paper folding. The remaining variables were all around the average. In general, Mathematics teachers had shortage of resources to implement visualization techniques in teaching Mathematics.

The other issue examined was teachers' knowledge of Mathematics visualization techniques. To that effect, Table 2 presents descriptive statistics and One-Sample t-values.

Table 2. Mean, SD and One-Sample t-values of Mathematics Teachers' Knowledge (MTK) of Mathematics Visualization Techniques (N = 183)

No	Variables	Mean	SD	t
	I have knowledge of:			
1	applying ***concept map*** in teaching Mathematics	3.93	.74	-1.30
2	presenting ***application of Mathematics topic/unit/chapter*** at the beginning of each topic/unit/chapter	4.12	.92	1.76
3	giving **practical application of Mathematics** in terms of projects that are collected from the field	3.30	1.00	-9.53*
4	giving Mathematics ***application by examples*** under each topics in the class	4.21	.92	3.05*
5	explaining mathematical concepts using ***non-examples*** in my teaching	2.65	1.25	-14.6*
6	explaining mathematical concepts using ***counter examples***	4.16	.85	2.61*
7	explaining mathematical concepts using ***compare and contrast***	3.91	.92	-1.28
8	giving ***reasoning*** in my explanation of mathematical concepts	4.15	.87	2.30*
9	***experimenting*** mathematical concepts	3.19	1.13	-9.68*
10	explaining mathematical concepts using ***real objects***	3.82	.97	-2.52*
11	applying manipulatives such as ***models*** that represent real objects	3.73	.97	-3.75*
12	using ***paper folding*** in Mathematics	3.53	1.01	-6.30*
13	applying ***graphic and pictorial*** presentations in Mathematics	3.94	.96	-.85
14	applying ***animation or simulation*** using computer or software	2.34	1.30	-17.3*
	Knowledge of implementing visualization techniques	**3.64**	**.54**	-8.91*

* p< 0.5; * Note: The observed means of Teachers' Knowledge of Mathematics Visualization Techniques items were compared against a specified mean (Test value) = 4 (Agree)

Table 2 indicated that the aggregate average value of teachers' knowledge of visualization techniques was significantly below the specified mean 4 (Agree) ($M = 3.64$, $SD = .54$, $t = -8.91$, $p < .05$). This result could be traced to very low level of teachers' knowledge on

- applying animation or simulation using computer or software ($M = 2.34$, $SD = 1.30$, $t = -17.30$, $p < .05$);
- explaining mathematical concepts using non-examples ($M = 2.65$, $SD = 1.25$, $t = -14.63$, $p < .05$);
- giving practical application of Mathematics in terms of projects that are collected from the field ($M = 3.30$, $SD = 1.00$, $t = -9.53$, $p < .05$); and
- experimenting mathematical concepts ($M = 3.19$, $SD = 1.13$, $t = -9.68$, $p < .05$)

Mathematics teachers had knowledge (similar to the specified mean of 'Agree = 4') on

- applying concept map in teaching Mathematics ($M = 3.93$, $SD = .74$, $t = -1.30$, $p > .05$);
- presenting an application of Mathematics topic/unit/chapter at the beginning of each topic/unit/chapter ($M = 4.12$, $SD = .92$, $t = 1.76$, $p > .05$);
- explaining mathematical concepts using compare and contrast in teaching ($M = 3.91$, $SD = .92$, $t = -1.28$, $p > .05$); and
- applying graphic and pictorial presentations in Mathematics ($M = 3.94$, $SD = .96$, $t = -.85$, $p > .05$)

In addition, the teachers had significantly higher knowledge on

- teaching mathematical application using examples under each topics in the class (M = 4.21, SD = .92, t = 3.05, p < .05);
- explaining mathematical concepts using counter examples in their teaching (M = 4.16, SD = .85, t = 2.61, p < .05); and
- giving reasons in their explanation of mathematical concepts (M = 4.15, SD = .87, t = 2.30, p < .05)

As regards Mathematics teachers' attitude towards teaching Mathematics using visualization techniques, Table 3 presents descriptive statistics and One-Sample t-values.

Table 3. Mean, SD and One-Sample t-values of Mathematics Teachers' Attitude (MTA) towards Teaching Mathematics using Visualization Techniques (VTs), (N = 183)

No	Variables	Mean	SD	t
1	I am not sure I have knowledge about VTs in teaching Mathematics	3.69	1.09	-3.79*
2	I feel confident in my ability of using VTs in teaching Mathematics	3.93	.99	-.90
3	I am sure that I can understand mathematical VTs	4.03	.84	.53
4	Mathematical VTs are hard for me	3.57	1.16	-4.97*
5	I doubt that I will be able to improve my teaching ability using VTs	2.91	1.21	-12.2*
6	I feel confidence while applying mathematical VTs	3.73	1.05	-3.44*
7	I am not sure of the parts of Mathematics in which VTs are implemented	3.68	1.08	-4.04*
8	I am sure I will improve my teaching using VTs	3.95	.95	-.78
9	I am not good in mathematical VTs	3.73	1.14	-3.17*
10	I am sure I can handle mathematical VTs well	3.80	.82	-3.24*
	Confidence in applying VTs in teaching Mathematics	3.70	.56	-7.22*
11	VTs help better ***understanding*** of mathematical concepts by students	4.30	.88	4.63*
12	VTs help students in **solving *problems of*** Mathematics	4.27	.82	4.43*
13	VTs improve the ***critical thinking*** of students in Mathematics	4.24	.75	4.32*
14	VTs enhance the ***reasoning power*** of students in Mathematics	4.05	.91	.82
15	VTs improve the ***creativity*** of students in Mathematics	4.15	.94	2.14*
16	VTs improve the ***achievement*** of students in Mathematics	4.11	.96	1.62

17	VTs increase ***social interaction*** between students	4.02	.92	.24
18	VTs enhance the ***close contact*** between teacher and students	3.66	1.12	-4.09*
19	VTs decrease the ***anxiety*** of students in learning Mathematics	3.52	1.08	-5.94*
20	VTs develop the ***confidence*** of students in learning Mathematics	4.24	.86	3.69*
21	VTs improve the ***attitude*** of students in learning Mathematics	4.19	.79	3.27*
22	VTs enhance the ***motivation*** of students in learning Mathematics	4.15	.88	2.35*
	Usefulness of Mathematics VTs in teaching Mathematics	4.08	.65	1.59
23	I enjoy going beyond the assigned work using mathematical VTs	3.85	.91	-2.27*
24	Mathematical VTs are enjoyable and stimulating to me	4.02	.92	.24
25	Mathematics is boring using mathematical VTs	3.72	1.22	-3.08*
26	I am not interested and willing to use mathematical VTs outside school	3.72	1.13	-3.35*
27	I am interested and willing to acquire more knowledge about mathematical VTs	4.15	.88	2.36*
28	I have always enjoyed teaching Mathematics using VTs	3.81	.94	-2.74*
29	Mathematical VTs make me feel uncomfortable and nervous	3.73	1.25	-2.96*
30	I have never liked mathematical VTs	4.00	1.12	.00
31	I would not like to develop my mathematical VTs skills	4.14	1.02	1.88
32	Mathematics is enjoyable using mathematical VTs	4.21	.74	3.92*
	Enjoyment in using Mathematics VTs in teaching Mathematics	3.94	.59	-1.50
	Attitude of teachers towards teaching Mathematics using VTs	3.93	.48	-1.68

* p< 0.5;
* Note: The observed means of Mathematics Teachers' Attitude towards Mathematics Visualization Techniques items were compared against a specified mean (Test value) = 4 (Agree)

Table 3 indicated that the aggregate average value of teachers' attitude towards teaching Mathematics using visualization techniques was positive and similar to the specified mean 4 (Agree) (M = 3.93, SD = .48, t = -1.68, p > .05). This result is attributable to the positive responses of Mathematics teachers on

- the usefulness of visualization techniques in teaching Mathematics (M = 4.08, SD =.65, t = 1.59, p > .05); and
- enjoying using Mathematics visualization techniques in teaching Mathematics (M = 3.94, SD = .59, t = -1.50, p > .05)

Some of the responses on the usefulness of Mathematics visualization techniques in teaching Mathematics were significantly higher than the specified mean of ‘Agree = 4’. These agreed that Mathematics visualization techniques

- help students understand concepts better ($M = 4.30$, $SD = .88$, $t = 4.63$, $p < .05$);
- help students solve mathematical problems ($M = 4.27$, $SD = .82$, $t = 4.43$, $p < .05$);
- improve the critical thinking abilities of students ($M = 4.24$, $SD = .75$, $t = 4.32$, $p < .05$),
- improve the creativity of students ($M = 4.15$, $SD = .94$, $t = 2.14$, $p < .05$)and
- improve the attitude of students towards learning Mathematics ($M = 4.19$, $SD = .79$, $t = 3.27$, $p < .05$)
- enhance the confidence of students in learning Mathematics ($M = 4.24$, $SD = .86$, $t = 3.69$, $p < .05$) and
- enhance the motivation of students in learning Mathematics ($M = 4.24$, $SD = .86$, $t = 3.69$, $p < .05$)

In addition, with regard to enjoying using Mathematics visualization techniques in teaching Mathematics, responses which were significantly higher than the specified mean of ‘Agree = 4’ were

- interest and willingness to acquire further knowledge of mathematical VTs ($M = 4.15$, $SD = .88$, $t = 2.36$, $p < .05$); and
- enjoying Mathematics while using mathematical VTs ($M = 4.21$, $SD = .74$, $t = 3.92$, $p < .05$).

The average value of teachers’ confidence in applying Mathematics visualization techniques in teaching Mathematics was slightly significantly below the specified mean of 4 (Agree) (M = 3.70, SD = .56, t = -7.22, p < .05). The challenges specified by teachers were

- their difficulty to use mathematical VTs
- their doubt to improve their teaching ability using VTs and
- decrease in the motivation of students to learn Mathematics

As regards the approaches teachers used to develop students' visualization in Mathematics, Table 4 presents descriptive statistics and One-Sample t-values.

Table 4. Mean, SD and One-Sample t-values for Approaches Mathematics Teachers employed to Develop Students' Mathematics Visualization **(**MTDSMV), (N = 183)

No	Variables	Mean	SD	t
I design my lesson to use:				
1	*concept maps, graphics/pictorial presentations* to clarify concepts	3.29	.96	-10.0*
2	*real-life applications* at the beginning, middle & end of lessons	3.27	1.03	-9.60*
3	*examples, non-examples and counter examples* to clarify concepts	3.66	.96	-4.79*
4	*comparing and contrasting, and reasoning* as central parts to clarify concepts	3.41	1.08	-7.43*
5	*experimentation and manipulatives* in explaining concepts	2.93	1.03	-14.0*
6	*animation/simulation/video/computer application* for some concepts	2.02	1.19	-22.6*
	Designing Mathematics lessons using visualization techniques	**3.10**	**.71**	-17.2*
In my teaching, I				
7	apply *concept map* in explaining mathematical concepts	3.38	.86	-9.79*
8	describe Mathematics concepts using *graphics and pictorial presentations*	3.72	1.03	-3.66*
9	present the *application of mathematical topics* at the beginning of teaching	3.60	1.02	-5.28*
10	show how to apply *real-life applications* as modelling in the field	3.04	1.12	-11.6*
11	use *examples* to explain mathematical concepts in class	4.23	.81	3.82*
12	use *non-examples and counter examples* in explaining concepts	3.08	1.20	-10.3*
13	use *compare and contrast* in explaining mathematical concepts	3.50	1.04	-6.49*
14	provide *reasons* in explaining mathematical concepts	3.78	.98	-3.01*
15	use *experimentation* for clarifying some concepts of Mathematics	2.84	1.12	-14.1*
16	use *real objects or models or paper folding* in teaching Mathematics	3.33	1.06	-8.50*
17	use *animation/simulation/video/computer application*	1.81	1.10	-26.9*
	Teaching mathematical concepts using visualization techniques	**3.30**	**.62**	-15.3*

To develop students' understanding and skills, I give students				
18	I activities for them to produce *concept maps* to see mathematical relationships	3.43	.99	-7.88*
19	problems in terms of *graphs and pictures* to exercise mathematical concepts	3.42	1.00	-7.94*
20	I assignments in the *real-life application* of Mathematics in terms of projects	3.24	1.12	-9.15*
21	*problems* to exercise mathematical concepts	3.87	.90	-1.98*
22	problems to compare & contrast and give reasons for mathematical steps	3.64	.94	-5.13*
23	assignments for them to produce *models* as representations of real objects	3.35	1.07	-8.24*
24	problems to *experiment* or use *paper-folding* in understanding concepts	3.16	1.03	-11.0*
25	problems to practice *animation/simulation/video/computer application*	1.85	1.17	-24.8*
	Exposing students to practice the visualization of mathematical concepts	**3.25**	**.68**	-15.0*
In order to assess students' progress, I ask them to				
26	relate mathematical concepts by using *concept map*	3.24	1.03	-9.97*
27	indicate the application of mathematical concept using *graphs or pictures*	3.08	1.05	-11.8*
28	clarify concepts using *real-life application*	3.14	1.03	-11.3*
29	give *examples of* mathematical concepts	3.29	1.01	-9.57*
30	give *non-examples of* mathematical concepts	3.07	1.04	-12.1*
31	*give counter examples for* false mathematical statements	3.51	.94	-7.02*
32	*compare and contrast and give reasons* for Mathematics concepts	3.26	.99	-10.1*
33	*experiment or use paper folding* for proofing Mathematics principles	2.96	1.04	-13.6*
34	clarify mathematical concepts by using *animation/simulation/video*	1.87	1.15	-25.0*
	Assessing students' Progress on visualizing mathematical concepts	**3.05**	**.65**	-19.9*
	Developing students' visualization in Mathematics	**3.19**	**.61**	-18.1*

* p< 0.5; Note: The observed means of Mathematics Teachers' Knowledge in Mathematics Visualization Techniques items were compared against a specified mean (Test value) = 4 (Quite often)

Table 4 indicates that the aggregate average value for developing students' Mathematics visualization was significantly below the specified mean 4 (implemented quite often) (M = 3.19, SD = .61, t = -18.14, p < .05). This result is significantly lower than the expected value of Mathematics teachers'

- designing lessons using visualization techniques (M = 3.10, SD = .71, t = -17.18, p< .05);

- teaching concepts using visualization techniques ($M = 3.30$, $SD = .62$, $t = -15.3$, $p < .05$);
- exposing students to practice the visualization of concepts ($M = 3.25$, $SD = .68$, $t = -15.0$, $p < .05$); and
- assessing students' progress on visualizing concepts ($M=3.05$, $SD=.65$, $t=-19.9$, $p < .05$)

The value for Mathematics' teachers use of examples to explain mathematical concepts in the class was significantly higher than the specified mean of 'Implemented quite often = 4' ($M = 4.23$, $SD = .81$, $t = 3.82$, $p < .05$). However, this result is significantly lower than the specified mean value of Mathematics teachers' designing their lessons to use

- experimentation and manipulatives in explaining concepts ($M= 2.93$, $SD= 1.03$, $t = -14.0$, $p< .05$) and
- animation/simulation/video/computer application for some appropriate concepts ($M = 2.02$, $SD = 1.19$, $t = -22.6$, $p < .05$); and

When it comes to teaching, Mathematics teachers used in their teaching

- experimentation for clarifying some concepts of Mathematics ($M = 2.84$, $SD = 1.12$, $t = -14.1 p < .05$); and
- animation/simulation/video/computer application ($M = 1.81$, $SD = 1.10$, $t = -26.9$, $p < .05$)

In addition, the values are significantly very lower than the specified mean of Mathematics teachers'

- giving problems for students to practice animation/simulation/video/computer application ($M = 1.85$, $SD = 1.17$, $t = -24.8$, $p < .05$); and
- assessing students' progress by using experiment or paper folding for proving principles ($M= 2.96$, $SD= 1.04$, $t= -13.6$, $p < .05$), and
- assessing students' progress by using animation/simulation/video in order to clarify concepts ($M=1.87$, $SD=1.15$, $t= -25.0$, $p < .05$).

On the whole, there are problems of using concept map, graphics/pictorial presentations, real-life applications, non-examples and counter examples, comparing & contrasting, and reasoning as central parts for designing lessons, teaching, exposing students to practice, and assessing students' progress on visualizing mathematical concepts.

The second research question was about the relationships between the availability of resources, Mathematics teachers' knowledge and attitude towards teaching using visualization techniques, and developing students' Mathematics visualization. Table 5 below presents the association between teachers' knowledge (MTK) and attitude towards visualization techniques (MTA), availability of resources (AR) and developing students' Mathematics visualization (MTDSMV).

Outcomes suggest that there were significant and positive associations between all variables except that between AR and MTA which was negligible (.086). The association between the dependent variable and its component variables of MTDSMV, DML, TMC, ESP, and ASP were all much larger than the medium with reference to Cohen's (1988) guidelines.

- developing students' visualization in Mathematics,
- designing Mathematics lessons using visualization techniques,
- teaching mathematical concepts using visualization techniques,

- exposing students to practice the visualization of mathematical concepts, and
- assessing students' on visualizing mathematical concepts

Table 5. Association between the variables MTK, MTA, AR and MTDSMV

Variables	1	2	3	4	5	6	7	8	Mean	SD
1. MTK	1								3.64	.54
2. MTA	.474*	1							3.93	.48
3. AR	.223*	.086	1						2.47	.83
4. MTDSMV	.472*	.288*	.539*	1					3.19	.61
5. DML	.382*	.217*	.484*	.904*	1				3.10	.71
6. TMC	.486*	.364*	.529*	.939*	.818*	1			3.30	.62
7. ESP	.465*	.238*	.453*	.929*	.768*	.835*	1		3.25	.68
8. ASP	.396*	.217*	.519*	.925*	.806*	.789*	.828*	1	3.05	.65

* $p < 0.5$;
MTK- Mathematics teachers' knowledge on Mathematics visualization;
MTA- Mathematics teachers' attitude towards Mathematics visualization;
AR - Availability of resources for employing Mathematics visualization;
DML - Designing Mathematics lessons using visualization techniques;
TMC-Teaching mathematical concepts using visualization techniques;
ESP – Exposing students to practice the visualization of mathematical concepts;
ASP – Assessing students' progress on visualizing mathematical concepts.

The associations between availability of resources for mathematical visualization techniques and the variables were much larger than the typical when reference is made Cohen's (1988) guidelines. Similarly, the association between Mathematics teachers' knowledge on visualization techniques with the variables of Mathematics teachers' attitudes towards were much larger when reference is made to Cohen's (1988) guidelines. For the other variables, the association was medium or typical.

- teaching Mathematics using visualization techniques,
- developing students' visualization in Mathematics,
- teaching mathematical concepts using visualization techniques,
- exposing students to practice the visualization of mathematical concepts

The third research question was about the contributions of availability of resources, teachers' knowledge and attitude towards employing visualization techniques on developing students' Mathematics visualization. To determine the extent of contributions to the variation in developing students' visualization, multiple regression was used. Table 6 below indicates the contribution of the independent variables to the variation in developing students' visualization in Mathematics and its components.

The multiple correlations (R) of all variables (MTK, MTA and AR) with developing students' visualization in Mathematics yield a value of 0.654 and the coefficient of determination was 0.428. This indicates that it is only about 43% of the variance in developing students' visualization that was contributed by the three variables. Of them, the availability of resources was a significant highest predictor variable since it explained about 24% of the variance (t= 7.900, p< 0.05) followed by teachers' knowledge which explained about 15% of the variance (t= 4.961, p< 0.05). The contribution of the attitude of teachers which stood at about 3% was insignificant (t= 1.467, P> 0.05). Such an outcome suggests that the development of visualization in students is more a result of availability of resources and knowledge of teachers.

Table 6. Contribution of the independent variables to developing students' visualization

Developing Students' Mathematical Visualization										
Variables	**r**	**B**	**SE**	**β**	**Contribution** (rβ×100%)	**t**	**R**	**R^2**	**adj R^2**	**F**
1. MTK	.472	.364	.073	.326	15.39%	4.961*	.654	.428	.418	44.63*
2. MTA	.288	.118	.081	.094	2.71%	1.467				
3. AR	.539	.336	.043	.458	24.69%	7.900*				
Designing Mathematics Lessons Using Visualization Techniques										
Variables	**r**	**B**	**SE**	**β**	**Contribution** (rβ×100%)	**t**	**R**	**R^2**	**adj R^2**	**F**

1. MTK	.382	.341	.094	.261	9.97%	3.639*	.562	.316	.305	27.59*
2. MTA	.217	.084	.103	.057	1.24%	.809				
3. AR	.484	.361	.054	.421	20.38%	6.644*				
Teaching Mathematical Concepts Using Visualization Techniques										
Variables	**r**	**B**	**SE**	**β**	**Contribution** (rβ×100%)	**t**	**R**	**R^2**	**adj R^2**	**F**
1. MTK	.486	.340	.073	.300	14.58%	4.652*	.670	.448	.439	48.48*
2. MTA	.364	.234	.081	.183	6.66%	2.907*				
3. AR	.529	.332	.042	.446	23.59%	7.830*				
Exposing Students to Practice Visualization of Mathematical Concepts										
Variables	**r**	**B**	**SE**	**β**	**Contribution** (rβ×100%)	**t**	**R**	**R^2**	**adj R^2**	**F**
1. MTK	.465	.460	.088	.368	17.11%	5.237*	.588	.345	.334	31.45*
2. MTA	.238	.045	.097	.032	.76%	.470				
3. AR	.453	.302	.051	.368	16.67%	5.929*				
Assessing Students' Progress on Visualizing Mathematical Concepts										
Variables	**r**	**B**	**SE**	**β**	**Contribution** (rβ×100%)	**t**	**R**	**R^2**	**adj R^2**	**F**
1. MTK	.396	.322	.083	.271	10.73%	3.892*	.595	.354	.343	32.70*
2. MTA	.217	.065	.091	.049	1.06%	.716				
3. AR	.519	.354	.048	.454	23.56%	7.373*				

* $p < 0.5$

Outcomes of the multiple correlations (R) among all variables under study (MTK, MTA and AR) with

- designing Mathematics lessons using visualization techniques and its coefficient of determination (0.562 and 0.316 respectively);
- teaching mathematical concepts using visualization techniques and its coefficient of determination (0.670 and 0.448 respectively);
- exposing students to practice the visualization of mathematical concepts and its coefficient of determination (0.588 and 0.345 respectively); and

- assessing students' on visualizing mathematical concepts and its coefficient of determination (0.595 and 0.354 respectively)

indicate that all variables in this study (MTK, MTA and AR) contributed

- 31.6% to designing Mathematics lessons using visualization techniques;
- 34.5% to exposing students to practice the visualization of mathematical concepts; and
- 35.4% to assessing students' visualization of mathematical concepts
- 44.8% to teaching mathematical concepts using visualization techniques.

The observed outcomes suggest that the highest contribution was made by teaching mathematical concepts using visualization techniques while the lowest was made by designing Mathematics lessons using visualization techniques.

The availability of resources for teaching mathematical visualization techniques was a significant highest predictor for

- designing Mathematics lessons using visualization techniques (20.38%);
- teaching mathematical concepts using visualization techniques (23.59%); and
- assessing students' on visualizing the mathematical concepts (23.56%).

Mathematics teachers' knowledge on visualization techniques contributed significantly to

- designing Mathematics lessons using visualization techniques (9.97%),
- teaching mathematical concepts using visualization techniques (14.58%),
- exposing students to practice the visualization of mathematical concepts (17.11%), and
- assessing students' on visualizing the mathematical concepts (10.73%).

Mathematics teachers' attitude towards visualization techniques contributed insignificantly to variations in

- designing Mathematics lessons using visualization techniques (1.24%,);
- exposing students to practice the visualization of mathematical concepts (.76%);
- assessing students' on visualizing mathematical concepts (1.06%).

Thus, the results of this study suggest that the availability of resources was the most dominant predictor for designing Mathematics lessons, teaching mathematical concepts, and assessing students' on visualizing mathematical concepts.

Discussion

Visualization techniques have lots of importance such as helping student learning at all levels (Stieff et al., 2007), engaging and allowing students to manipulate variables, setting the pace of interactions and improving the learning of students (Rapp, 2007). Further, students' discourses and social interactions are influenced by the resources available to them (Kozma, 2000). Outcomes suggested that teachers faced shortage of resources to implement visualization techniques in teaching Mathematics. More serious problems were related to shortage of computers, software, videos, internet access, and real objects or models/kites which are useful to clarify concepts. This is in line with Becta (2004) who claimed that one of the barriers for teaching Mathematics using visualization is lack of access to resources.

In the Ethiopian context, only 33% of primary schools have access to electricity which makes it difficult to implement animation/simulation/video/computer application in most of the schools. Beyond electricity, the big challenge in Ethiopia is access to other multimedia. As indicated by MOE (2017), 67% of primary schools have radios while 33% have tape recorders and 11% have video recorders. Of the primary schools in the country, only 45% have library while 22% have laboratory and 61% have pedagogical centres (MOE, 2016). These imply that there is a shortage of reference books and other supplies for using concept maps, graphics, real-life applications, non-examples, counter examples and compare & contrast. In addition, the shortage of laboratories and pedagogical centres also affects what teachers do to experiment and apply manipulatives in mathematical expressions.

When it comes to teachers, they need to have extensive background knowledge in order to select the right resources for the right teaching and learning objectives (Wellington, 2002), integrate them to their pedagogical content knowledge (Shulman, 1986), interpret visualizations (Linn, 2003), and, finally, improve students' learning (Butterworth, 1999). In relation to that, outcomes of this study also indicated that Mathematics teachers had knowledge about teaching Mathematics application by examples, explaining mathematical concepts using counter examples, and giving reasons in their explanation. However, they had shortage of knowledge on implementing Mathematics visualization techniques especially in applying animation or simulation using computer or software, explaining mathematical concepts using non-examples, giving practical application of Mathematics, and experimenting with mathematical concepts. With respect to computer usage, the outcome is in line with Jasute (2013) who asserted that most teachers failed to use computers for preparing and teaching their lessons.

Among the barriers for implementing mathematical visualization techniques are found lack of confidence and resistance to change (Becta, 2004). Lack of confidence is directly related to the lack of competence in integrating ICT into pedagogical practice (Becta, 2004). This study indicated that Mathematics teachers' attitude towards teaching Mathematics using visualization techniques, usefulness of Mathematics visualization techniques and enjoying using Mathematics visualization techniques in teaching Mathematics were positive and similar to the specified mean 4 (Agree). However, their confidence in applying Mathematics visualization techniques in teaching Mathematics was below the expected. The result is in line with the studies of Balanskat et al. (2006) who indicated that teachers had a positive perception of visualization tools, and Mayer (2001) who stated that teachers demonstrated a range of positive responses for science visualization programs. The result indicated that visualization techniques help students to better understand and solve problems in Mathematics while at the same time improving critical thinking and creativity.

In addition to the above, visualization techniques enhance the attitude of students towards learning Mathematics and improve their confidence and motivation to learn Mathematics. For teachers, Mathematics is enjoyable with visualization techniques and serves as a motivating factor to acquire further knowledge. Thus, in order to help students to develop visualization knowledge and skills, teachers and designers of curriculum materials should first be aware of the factors that influence students' choice of problem-solving methods and the processes and roles that visualization plays in mathematical problem solving (Yin, 2009). Besides, Linn (2003) claims that there should be proper planning as to when and how to use different types of visualization in order to maximize their usefulness for student learning

The current study suggested that the degree to which teachers' were able to develop students' visualization in Mathematics was below the expected level (implemented quite often). This could be attributed to the low level of implementation of Mathematics visualization techniques in designing lessons, applying visualization in teaching, exposing students to practice, and assessing students' progress. Mathematics teachers frequently used examples to explain mathematical concepts in class; yet, they rarely implemented them in designing lessons, teaching students, exposing students to practice, and assessing students' progress using animation/simulation/video/computer application, and using experimentation and manipulatives in explaining concepts.

In addition to the above, availability of resources and teachers' knowledge and attitude towards visualization techniques were associated with developing students' visualization in Mathematics. Of these, the unavailability of resources was the most dominant predictor variable followed by deficiency in teachers' knowledge on the area. The contribution of absence of a positive attitude to the development of the skill was insignificant.

Numerous studies and reports throughout the past decade showed that traditional Mathematics was ineffective in the modern classroom (NCTM, 2000). In traditional teaching of Mathematics, concepts are not integrated and taught to relate to the world. There has been apparent agreement among many educators that students need to understand not only procedural knowledge, but also conceptual knowledge in Mathematics (Cangelosi, 1996). Other researchers expressed that, in order to align research-based principles with curricula, students need experience in using Mathematics to solve real-life problems (NCTM, 2000). Experiments have also a great potential in Mathematics lessons to introduce the concept of variable (Michelsen, 2006), and manipulatives are effective tools in Mathematics education since they help students move from the concrete to the abstract level of understanding. Students who see, touch, sort, take part, and manipulate physical objects begin to develop clearer mental images and can represent abstract ideas more completely than those whose concrete experiences are limited (Heddens, 1986).

Conclusions

The purpose of this study was to assess the contributions of availability of resources, Mathematics teachers' knowledge, and attitude towards teaching using visualization techniques on developing students' visualization in Mathematics. Outcomes suggested that there is shortage of resources to implement mathematical visualization techniques. Besides, Mathematics teachers had insufficient knowledge to apply animation or simulation using computer software and explain mathematical concepts using non-examples. Moreover, they failed to show practical application of Mathematics and experimenting mathematical concepts. While they exhibited a positive attitude toward teaching Mathematics using visualization techniques, their confidence in applying visualization techniques and developing students' visualization in Mathematics remained below the expected.

Of the variables used in the study, the availability of resources and teachers'' knowledge were the two most dominant predictor variables for developing Mathematics visualization techniques. Thus, based on these results of the study, it may be concluded that visualization techniques were not effectively used to help students develop the techniques and achieve better results in Mathematics learning.

Recommendations

In order to develop students' Mathematics visualization, the first thing to be done is availing Mathematics visualization resources by schools for teachers to use them. It is also important to train teachers on how to apply visualization techniques. When designing their lessons, teachers should also select the type of visualization relevant and appropriate for the particular task, the scientific concept, and students' background knowledge and skills.

During teaching, teachers should combine visualization with verbal or textual information for conceptual understanding, and provide explicit explanations or guidance about the most relevant features and the application of the display. Teachers should also encourage students to practice visualization techniques by giving sufficient time as well as monitoring and assessing their progress.

Schools and department heads should make sure that teachers are sufficiently engaged in applying Mathematics visualization techniques into the teaching of Mathematics. The Mathematics curriculum and textbooks should be designed in order to properly and sufficiently apply visualization in the teaching-learning of Mathematics.

NOTES

1. http://math.unipa.it/~grim/Jhoffman.PDF
2. http://portaldoprofessor.mec.gov.br/storage/materiais/0000012853.pd
3. https://dera.ioe.ac.uk/1603/1/becta_2004_barrierstouptake_litrev.pdf

REFERENCES

Alias, M. (2000). *Spatial visualisation ability and problem solving in civil engineering: doctoral thesis*. Guilford: University of Surrey.

Alias, M., Black, T.R. & Gray, D.E. (2002). Effect of instruction on spatial visualisation ability in civil engineering students. *Int. Educ. J., 3*(1), 1–12.

Anderson-Inman, L. & Ditson, L. (1999). Computer-based concept mapping: a tool for negotiating meaning. *Learning & Leading Technology, 26*, 6-13.

Bransford, J.D., Brown, A.L. & Cocking, R.R. (2000). *How people learn: brain, mind, experience, and school.* Washington: National Academy Press.

Butterworth, B. (1999). *The mathematical brain*. London: Macmillan.

Cangelosi, J.S. (1996). *Teaching mathematics in secondary and middle school: an interactive approach*. Englewood Cliffs: Prentice Hall.

Cifuentes, L. & Hsieh, Y.J. (2003). Visualization for construction of meaning during study time: a quantitative analysis. *Int. J. Instruct. Media, 30*(3), 263–273.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* Hillsdale: Lawrence Erlbaum Associates.

Heddens, J.W. (1986). Bridging the gap between the concrete and the abstract. *Arithmetic Teacher, 33*(6), 14–17.

Jasute, E. (2013). Research on interactive geometry visualization for secondary school. *World Conference on Computers in Education,* July 2-5, Toruń, Poland.

Kadijević, D. (1999). Conceptual tasks in mathematics education. *Teaching Math., 2*(1), 59-64.

Konyalioglu, A.C. (2019). *Investigation of effectiveness of visualization approach on understanding of concepts in vector spaces at the university level: doctoral dissertation*. Erzurum: Atatürk University.

Konyalioglu, A.C., Ipek, A.S. & Isik, A. (2003). On the teaching linear algebra at the university level: the role of visualization in the teaching vector spaces. *J. Korean Soc. Math. Educ. D, 7*(1), 59-67.

Linn, M C. (2003). Technology and science education: starting points, research programs, and trends. *Int. J. Sci. Educ., 25,* 727–758.

Marzano, R.J., Pickering, D.J. & Pollock, J.E. (2001). *Classroom instruction that works: research-based strategies for increasing student achievement*. Washington: ASCD.

Mayer, R (2001). *Multimedia learning.* Cambridge: Cambridge University Press.

Michael H. (2002). Computer simulations and modelling in mathematics education. *Proc. Int. Conf. Humanistic Renaissance Mathematics Education,* pp. 176-180.

Michelsen, C. (2006): Functions: a modelling tool in mathematics and science. Z. Didaktik Mathematik *(ZDM), 38*, 269-280.

MOE [Ministry of Education] (2001). *Mathematics syllabus.* Singapore: Ministry of Education.

MOE [Ministry of Education] (2007). *Mathematics syllabus primary.* Singapore: Ministry of Education.

MOE [Ministry of Education] (2016). *Education statistics: annual abstract for 2015/2016*. Singapore: Ministry of Education.

MOE [Ministry of Education] (2017). *Education statistics: annual abstract for 2016/2017*. Singapore: Ministry of Education.

NCTM [National Council of Teachers of Mathematics] (2000). *Principles and standards for school mathematics*. Reston: NCTM.

Osborne, J. & Hennessy, S. (2003). *Literature review in science education and the role of ICT: promise, problems and future directions.* London: Futurelab.

Phillips, L.M., Norris, S.P. & Macnab, J.S. (2010). *Visualization in mathematics, reading and science education*. Dordrecht: Springer.

Raphael, D. & Wahlstrom, M. (1989). The influence of instructional aids on mathematics achievement. *J. Res. Math. Educ., 20(2),* 173-190.

Rapp, D.N. (2007). Mental models: theoretical issues for visualizations in science education (pp. 43-60). In: Gilbert, J.K. (Ed.). *Visualization in science education*. Dordrecht: Springer.

Rieber, L.P. (1990). Using computer animated graphics in science instruction with children. *J. Educ. Psych., 82(1),* 135–140.

Rösken, B. & Rolka, K. (2006). A picture is worth a 1000 words – the role of visualization in mathematics learning (pp. 457-464). In: Novotná, J., Moraová, H., Klátká, M. & Stehlíková, N. (Eds.). *Proc. 30th Conf. Int. Group Psychology Mathematics Education, vol. 4*. Prague: PME.

Sezer, R. (2010). Pulling out all the stops. *Education*, *130*, 416-423.

Shulman, L.S. (1986). Those who understand: knowledge growth in teaching**.** *Educational Researcher, 15*(2), 4-14.

Stief, M., Bateman Jr., RC. & Uttal, D.H. (2007). Teaching and learning with three-dimensional representations (pp. 93-120). In: Gilbert, J.K. (Ed.). *Visualization in science education*. Dordrecht: Springer.

Tufte, E.R. (1990). *Envisioning information*. Cheshire: Graphics Press.

Webb, M.E. (2005). Affordances of ICT in science learning: implications for an integrated pedagogy. *Int. J. Sci. Educ.*, *27*, 705-735.

Webb, M.E. (2010). Technology-mediated learning (pp. 158-182). In: Osborne, J. & Dillon J. (Eds.). *Good practice in science teaching -what research as to say*. Maidenhead: Open University Press.

Wellington, J.J. (2002). *Teaching and learning secondary science: contemporary issues and practical approaches*. London: Taylor & Francis.

Wu, H.-K. & Shah, P. (2004). Exploring visuospatial thinking in chemistry learning. *Sci. Educ.*, *88*, 465–492.

Yanuarto, W.N. (2016). Students' awareness on example and non-example learning in geometry class. *Int. Electronic J. Math. Educ.*, *11*, 3511-3519.

Yin, H.S. (2009). *Visualization in primary school mathematics: its roles and processes in mathematical problem solving: PhD thesis*. Singapore: Nanyang Technological University.

✉ Dr. Demis Zergaw (corresponding author)
College of Education and Behavioral Studies
Addis Abbaba University
Addis Abbaba, Ethiopia
E-Mail: demiszergaw75@gmail.com